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THE

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A COLLECTION OF EXAMPLES

COMPOSED AND ARRANGED ON A NEW PLAN.

FOR THE USE CHIEFLY OF

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BY

W. H. WINGATE.

LONDON:
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1865.


BIBLIOTHECA ROMANA

PREFACE.

WORKS on ARITHMETIC are so numerous, and, generally speaking, so complete and well adapted to all classes of learners, that an additional one on the subject would seem to be absolutely superfluous. Certainly it should bring with it some new if only trifling pretension to notice, or it is likely to be consigned to neglect and disuse—the appropriate fate of an unnecessary book.

It cannot be denied by professors of experience that the teaching of Arithmetic is one of the most tedious as well as irregular of their occupations. The class system fails in most schools on account of irregular attendance, or difference in ability among the pupils, some remaining hopelessly behind the others. Teaching separately appears to be the general method, and to facilitate this should be the object of any new work on the subject. A difference of opinion exists with regard to the answers of the questions; in some books they are inserted with the questions—in others they are either omitted, or placed collectively at the end. It seems contrary to common sense that a boy should know the result before he has worked the operation,

and on the other hand, if the answers are not given, much *valuable* time is lost by the teacher in referring to the key, often several times for the same operation. This inconvenience may be avoided by encouraging the pupil to prove his own work.

The ordinary way of proving is by a reverse operation, in which the proof depends on the first calculation; as, for instance, proving division by multiplication, or in reduction by multiplying the last result by the divisors or *vice versâ*. These methods are often fallacious, as the divisors or multipliers may be wrongly taken, and the proof be simply an inversion of errors. It will also be admitted that boys show, in general, an unwillingness to prove their work, for the simple reason that, suspecting the first operation to be incorrect, they know the proof will be a useless operation. The plan I have adopted in this work may be called the Duplicate or Parallel system, each example being composed of two questions involving different figuring, with the exception of the results which should come out alike. This plan has been adopted by the author in practice, and he has invariably found that great interest has been evinced by the pupil in working the proof sum, and a greater satisfaction experienced than when ordinary Arithmetic books have been used. In this lies the pretension of this Manual to novelty, and it is hoped to utility, in imparting the knowledge of a science which is found to be indispensably necessary in the commonest education.

It will be noticed that some less useful rules, as

Alligation, Position, &c., have been omitted. Exchange has not been treated at length, as it is seldom required of youths, and is better acquired by actual practice in business. In all sums consisting merely of division, the remainders should be considered fractionally. Addition and Subtraction are proved by a double operation; and, although in the easier examples the process known by schoolboys as copying or fudging might be resorted to, it is thought that few will care to do that which a cursory glance of the teacher would inevitably discover.

Many of the proof sums are of the same rule as the first in each example, and it is hoped that they have been composed with sufficient ingenuity to render the figuring entirely different except in the result.

The Author will be glad to find that his simple method has facilitated the work of both master and pupil, in teaching and acquiring a science which may properly be called one of the most useful of any taught in schools.

CONTENTS.

SIGNS AND ABBREVIATIONS.

+ Plus, or more, as $8+6=14$

− Minus, or less, as $9-5=4$

× Multiplied by, as $5\times4=20$

÷ Divided by, as $8\div2=4$

= Equals, as $4\times6=24$

3^2 means that 3 is to be squared

4^3 means that 4 is to be cubed.

$\sqrt{16}$, or $16^{\frac{1}{2}}$, means that the sq. of root of 16 is to be taken.

$\sqrt[3]{27}$, or $27^{\frac{1}{3}}$, means that the cube root of 27 is to be taken.

i.e. that is; viz. namely;

cwt. hundred weight; qr. quarter; lb. pound;

£, *libræ*, pounds sterling; *s*, *solidi*, shillings;

d, *denarii*, pence; 5/6 means 5*s*. 6*d*.

7/9/64 means 7th day, 9th month, 64th year, or 7th Sept. 1864.

ARITHMETICAL TABLES.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Money Table.

Farthings	d.	Pence	s.	d.	Shillings	Pounds		
4	1	12	1	0	20	£1	0	0
8	2	20	1	8	30	1	10	0
12	3	24	2	0	40	2	0	0
16	4	30	2	6	50	2	10	0
20	5	36	3	0	60	3	0	0
24	6	40	3	4	70	3	10	0
28	7	48	4	0	80	4	0	0
32	8	50	4	2	90	4	10	0
36	9	60	5	0	100	5	0	0
40	10	70	5	10	200	10	0	0
44	11	72	6	0	300	15	0	0
48	12	80	6	8	400	20	0	0
		84	7	0	500	25	0	0
		90	7	6	600	30	0	0
		96	8	0	700	35	0	0
		100	8	4	800	40	0	0
		108	9	0	900	45	0	0
		110	9	2	1000	50	0	0
		120	10	0				
		130	10	10				
		132	11	0				
		140	11	8				
		144	12	0				

Troy Weight.

24 grains (grs.) . 1 pennyweight (dwt.)
20 dwts. . . . . 1 ounce (oz.)
12 ozs. . . . . 1 pound (lb.)

Apothecaries' Weight.

20 grains . . . 1 scruple (scr.)
3 scr. . . . . 1 dram (dr.)
8 drs. . . . . 1 ounce (oz.)
12 ozs. . . . . 1 lb.

Avoirdupois Weight.

16 drams . . . 1 ounce
16 ozs. . . . . 1 lb.
28 lbs. . . . . 1 quarter (qr.)
4 qrs. . . . . 1 hundredweight. (cwt.)
20 cwt. . . . . 1 ton

Cloth Measure.

2¼ inches . . . 1 nail
4 nails . . . . 1 quarter
4 quarters . . . 1 yard (yd.)

Liquid Measure, for wine, spirits, beer, &c., &c.

2 pints (pts.) . . 1 quart (qt.)
4 qts. . . . . 1 gallon

Dry Measure, for seeds, corn, fruit, &c., &c.

2 pints (pts.) . . 1 quart (qt.)
4 qts. . . . . 1 gallon
2 gallons . . . 1 peck (pk.)
4 pks. . . . . 1 bushel
8 bushels . . . 1 quarter (qr.)
5 quarters . . . 1 load.

Long Measure.

12 inches . . . 1 foot (ft.)
3 ft. . . . . 1 yard (yd.)
5½ yds. . . . 1 pole or perch
40 perches . . . 1 furlong
8 furlongs . . . 1 mile
3 miles . . . . 1 league

Square Measure.

144 inches . . . 1 ft.
9 feet . . . . 1 yard
30¼ yds. . . . 1 pole
40 poles . . . 1 rood
4 roods . . . 1 acre
640 acres . . . 1 mile sq.

Cubic Measure.

1728 inches . . 1 foot
27 feet . . . 1 yard

Time Table.

60 seconds . . 1 minute
60 minutes . . 1 hour
24 hours . . . 1 day
7 days . . . 1 week
4 weeks . . . 1 month
52 weeks . . . 1 year
365¼ days nearly 1 year

Quarter Days.

Lady-day . . . . March 25
Midsummer-day . . June 24
Michaelmas-day . . Sept. 29
Christmas-day . . Dec. 25

STANDARDS.

The imperial standard of weight is the pound, and that of measure the yard and gallon. The imperial yard was fixed by Act of Parliament, 1824 (which came into operation in 1826), to be a certain part of the length of a pendulum vibrating every second in the latitude of London and in a vacuum; the length of such a pendulum was found to be 39·13929 inches, of which 36 were declared to be the yard. If the standard yard now kept in the Tower were lost, it could be easily and correctly replaced by scientific means. The standard lb. Troy, made at the same time, was considered to be composed of 5760 grains, and 7000 such grains were to be a pound avoirdupois. It was ascertained that a cubic inch of distilled water, weighed under certain atmospheric conditions, was equal in weight to 252·458 grs. It was also determined that the imperial gallon should contain 277·274 cubic inches; and a gallon of pure distilled water will, under certain atmospheric conditions, just weigh 10 lbs. (avoirdupois).

NOTES ON WEIGHTS AND MEASURES.

The word Troy may possibly be derived from Troyes in France, where large fairs were formerly held, and where it was introduced from Cairo about the time of the Crusades. Some suppose the term is derived from Trinobantium or a corruption of it. Gold, silver, precious stones, electuaries, &c., are sold by this weight. The fineness of gold is expressed by carats fine; thus 14 carats fine mean 14 carats pure gold, and 10 copper. A carat means the 24th part of the whole.

Standard or Mint Gold is 22 pts. pure and 2 copper alloy—the alloy hardens the pure metal, and renders the melting of the coin a loss in value

The weight of a sovereign should not be less than 5 dwts. 2⅔ grs. allowing nearly 11 grs. for wear.

Coins formerly in Use.

Groat, 4*d.*; Tester, 6*d.*; Noble, 6*s.* 8*d.*; Angel, 10*s.*; Mark or Merk, 13*s.* 4*d.*; Carolus, 23*s.*; Jacobus, 25*s.*; and Moidore, 27*s.*

Avoirdupois Weight.—Avoirs, goods; poids, weight; or it may be derived from Avoir, to have, dupoids, some weight, in allusion to the new Troy lb. then considered too light.

1 lb. Avoir. is equal to 14 ozs. 11 dwts. 15½ grs. Troy.

10 lbs. Avoir. is the weight of a gallon of pure water.

The stone is a variable weight, in some counties being 14 and in others 16 lbs. In London 8 lbs. of meat make a stone.

Apothecaries' Weight. — The lb. Apoth. and lb. Troy are the same, but differently divided. Physicians make use of the following signs in writing their prescriptions: scruple, ℈; dram, ʒ; ounce, ℥; and pound, lb. Drugs are bought and sold by Avoirdupois weight.

Cloth Measure.—Cloth, linen, etc., are now sold only by the yard, and its fractional parts as quarters and eighths. The ells formerly used were the English ell = 5 qrs., the Flemish ell = 3 qrs., and the French ell = 6 qrs.

Long Measure, also called Lineal Measure.—The yard is the standard. This was formerly taken to be the length of the king's arm, but it is now scientifically determined. The word inch is derived from uncia, Lat. for twelfth part. Gunter's chain, used in measuring land, consists of 100 links, each 7·92 in. long; whole length, 66 feet.

Square Measure. — So called because a measurement is made on the square or two ways, i.e. by length and breadth. This measure is used to express the largeness of the surface only, without reference to thickness—it is thus is expressed the size of fields, &c, bricklayers' and painters' work, &c.

Liquid Measure.—The gallon is the standard. All spirits, malt liquors, oils, &c. &c., are sold by this measure. Besides the pint, quart, and gallon, there are the gill, used in some parts, and the quartern—the former being half-a-pint, and the latter, as the term implies, a quarter of a pint; and there are larger measures, such as pipes and butts; but it is usual to gauge the vessel containing the liquid, and express the contents in gallons, &c.

Dry Measure. — There are various terms used in this measure, according to the locality. In London and most country places, the term quarter is used to express the quantity of corn; in some parts, however, 'comb' is used, signifying 4 bushels. Dry measure is used for seeds, corn, nuts, fruit, &c.

TIME is the measure of change, for were there no change there could be no recorded time. There are two natural standards, the day and the year. The length of the day is a fixed and even quantity. The length of the year, however, is a more difficult measurement, and is different according to the mode of calculation or observation. The solar year consists of 365·242218 mean solar days = 365 days, 5 hours, 48 minutes, 47½ seconds nearly.

The common year consists of 365 days, and is nearly ¼ of a day short of the true solar year. Were this difference unnoticed, the names of the seasons would soon become misnomers, as summer would in course of time fall in the spring, and Christmas in the autumn. Julius Cæsar attempted to set the matter right by adding a whole day every fourth year. This arrangement was still liable to error, which accumulated every year, until, in 1582, the vernal equinox occurred on March 11 instead of March 21—that is, the calendar was behind time 10 days. Pope Gregory ordered 10 days to be omitted in 1582, and October 4 of that year was called October 15. He further ordered, to prevent future error, that every four hundredth year, reckoning from 1600 only, should be leap year, and not the intervening centuries.

The New Style was introduced into England 1751, and it was then found necessary to omit 11 days.

Peculiar Measures.

Fother of lead	19½ cwt.
1 stone (butcher's meat)	8 lbs.
A firkin of butter	56 lbs.
A „ soap	94 „
A barrel of anchovies	30 „
A „ raisins	112 „
A „ soap	256 „
A fathom	6 feet
A clove of wool	7 lbs.
A stone „	14 „
A tod „	28 „
A wey „	182 „
A pack „	240 „
A sack „	364 „
A last	4368 „
A truss of straw	36 „
A „ new hay	60 „
A „ old hay	56 „
A load of hay or straw	36 trusses
Great lb. of silk	24 ozs.
A yard of land	30 acres
A hide „	100 acres
A cubit „	18 inches
A pace	5 feet
A degree	69⅓ miles
Anker (wine or spirits)	10 gallons
Runlet „	18 „
Tierce „	42 „
Hogshead	63 „
Puncheon	85 „
Pipe	126 „
Firkin (beer)	9 „
Kilderkin	18 „
Barrel	36 „
Hogshead	54 „
Butt	108 „
Strike (corn)	2 pecks
Chaldron	32 bushels
Last	10 quarters

Thirty days hath September,
April, June, and November,
February hath twenty-eight alone,
All the rest have thirty-one,
Except in leap year, at which time,
February's days are twenty-n

ARITHMETIC.

ARITHMETIC is the science of ordinary numerical calculation, and is divided into several parts, of which the elementary are Numeration, Addition, Subtraction, Multiplication, and Division.

NUMERATION.

Numeration teaches to express orally, or in writing, the value of any number given in figures; thus 27 is called twenty-seven, and in the same manner, numbers given in words are expressed in figures, as five hundred and nine, 509. The extreme right hand figure of any number is called units, the next tens, &c., in the following order, which should be committed to memory: Units (1), tens (2), hundreds (3), thousands (4), tens of thousands (5), hundreds of thousands (6), millions (7), tens of millions (8), hundreds of millions (9), &c. Thus we learn that to write a million we require seven figures. Ordinary numbers are seldom expressed in words higher than hundreds of millions, but they may be carried on to billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, each of the above denominations requiring six extra figures to express it.

EXAMPLES.

Write in words—27; 189; 4040; 71865; 26189; 3170190; 21361546; 713819136; 181300; 21010126.

Write in figures the following quantities: forty-five; three hundred and nine; one thousand two hundred and sixteen; thirty-five thousand nine hundred and twenty-nine; four hundred and seventy-three thousand nine hundred and sixty-one; one million ten thousand and six; five thousand and eleven; seventy-nine; seven millions eight hundred and six thousand; nine hundred and forty; seven hundred and twenty-nine millions; five hundred and forty-four thousand and twenty-nine.

ADDITION.

Addition, from the Latin (addo), teaches to find how many in all are contained in several numbers. Place the numbers to be added under each other, observing to place units under units, &c.; draw a line to separate the result from the given numbers. Add the lowest right hand figure to the one immediately above it; to this sum add the next figure, and so on till the top one has been included; put under the column you have just added the last figure of the number which has resulted from the addition, and carry the others to the next column, which add in the same way, and so on to the extreme left hand column, the total of which is put down, and the addition is complete.

EXAMPLE.

3792
2416
5137
6249

In working the example given, say 9 and 7 are 16, and 6 are 22, and 2 are 24; put down 4 and carry 2 (that is, add 2 to the lowest figure of the next column), then say 2 and 4 are 6, and 3 are 9, and 1 are 10, and 9 are 19; put down 9 and carry 1 to the next column, and so on to the last column, the total of which (17) is put down, and the addition is complete.

The work may be tested by adding downwards, or by adding all but the top line, and then adding the top line to this total for the proof.

Exercise 1.

(1)

2734	3794
4163	2168
5236	4375
2347	4143

(2)

64538	51436
79625	92727
81364	65245
59189	75308

(3)

989769	897778
516342	361248
483658	638752
597326	269137
402674	730863
896756	988747

(4)

341526	714328
658474	285672
796389	787273
989769	998885
526143	614325
473857	385675

(5)

3959076	3591112
2065368	1444992
1974632	2555008
2389304	1076548
1610696	2963452
3587024	3954988

(6)

79163847	64183625
20836153	35816375
34761986	79163458
65238014	20836542
73816357	79138162
26183643	20861838
98976348	89354326
89765437	99387459

Exercise 2.

(1) Add together twenty-nine, three thousand seven hundred and fifty-four, seventy-nine, one hundred and sixty-nine thousand, seven hundred and fifty-one, ten thousand and thirteen, twenty-seven millions, one hundred and forty-one thousand six hundred and thirty-five, and fifty-eight.

Add together sixteen, three thousand seven hundred and sixty-seven, ten thousand and twenty-six, sixty-six, one hundred and sixty-nine thousand seven hundred and fifty-one, 13570817, 13570829, and forty-seven.

(2) Add together 372, 41635, 29, 1010, 2618, 153169, 473, and nine hundred and seventy-six.

Add together 261, 41746, 60, 977, 2608, 153179, 595, and nine hun-hundred and fifty-four.

(3) If A pays me 793*l.*, B 117*l.*, C 2463*l*, and D 3927*l.*; how much do they pay me in all?

I paid one merchant 682*l.*, another 228*l.*, another 1352*l.*, and another 5038*l.*; how much did I pay them in all?

(4) There 297364 inhabitants in one town, 35162 in a second, 319264 in a third, 416279 in a fourth, 893796 in a fifth, and 327963 in a sixth; how many are there in all?

There are 75142 yards of cloth in one warehouse, 257384 in a second, 652597 in a third, 82946 in a fourth, in a fifth 782685, and in a sixth 439074; how many are there in all?

SUBTRACTION.

Subtraction (from the Latin sub, traho,) enables us to find out how much one number is greater than another; the excess is called the difference, or remainder; thus, if I take 7 from 9, 2 remains; the upper or larger number is called the minuend, because it is to be lessened (from the Latin minuo, to lessen), and the smaller number is called the subtrahend, because it is to be taken away from the other. To prove the operation, add the difference to the less and it will give the greater number.

Operation.—Begin, as in addition, at the lower right hand figure, and take it from the one above it; if the upper figure is less than the lower figure, add 10 to it, and then substract, putting down the difference under the lower figure; if you add 10 to an upper figure you must add 1 to the next lower before taking it from its corresponding upper figure.

EXAMPLE.

3749
1386
2363

In working the example given, say 6 from 9 leaves 3; put down 3; 8 from 4 cannot be taken; borrow 10, then 8 from 14, remain 6; now, as you borrowed 10 to add to 4, you must add 1 to the 3, which is the nearest lower figure; then 4 from 7 remain 3, and 1 from 3 remain 2, and the subtraction is complete.

EXERCISE 3.

In the following examples, prove the first operation by the double subtraction; that is by subtracting the two

smaller quantities successively from the larger in the second. part of each example.

(1)

79163428	86073461
23591647	14602754
.....	.....
.....	15898926
	.....

(2)

95461738	81017294
29798396	12866557
.....	....
.....	2487395
	.....

(3)

813526193	1369081748
398679386	778845665
.....	.....
.....	175389276
	.....

(4)

76143862	165032750
29897968	86597579
.....	.....
.....	32189277
	.....

(5)

539214 0	142810528
7675746	58669765
.....	.....
.....	37894869
	.....

Exercise 4.

(1) From 7964 take one thousand and fifty-five, and from 9075 take two thousand one hundred and sixty-six.

(2) What is the difference between seven hundred and ninety-five and 264? also between 999 and four hundred and sixty-eight?

(3) Which is the greater: the sum of 8082, 2875 and 105, or the sum of 9047, 1822 and 121? Also take the sum of 8082, 2875 and 105 from the sum of 9047, 1822 and 121.

(4) From nine hundred and sixty millions, seven hundred and forty one thousand nine hundred and six take 713793642.

From 849630795, take six hundred and two millions, six hundred and eighty-two thousand, five hundred and thirty-one.

(5) Paid away 271*l.* out of 947*l.*, and 382*l.* out of 1058*l.*, how much remains?

(6) Bought 79634 yards of calico, and sold at different times the following quantities—2956 yds., 713 yds., 8193 yds., 106 yds., 17 yds., and 1015 yds., how much remains?

One merchant has in stock 80633 yds. of ribbon, of which the following quantities are of inferior quality—2912 yds., 757 yds., 8299 yds., 1032 yds., and 999 yds.; how many of good quality are there?

(7) An army of seventy-nine thousand, nine hundred and ten men went to battle, 5089 were killed, 10379 wounded, 2935 taken prisoners, and 1376 ran away; how many men are left in fighting order?

A gentleman has a fortune left him of 93470*l.*; he laid out in the purchase of a house, 12350*l.*, furniture, 3764*l.*, horses, 2050*l.*, carriages, 950*l.*, clothes, 574*l.*, and jewellery, 8976*l.*, and sundry other things 4675*l.*; how much has he still left?

MULTIPLICATION.

Multiplication, from the Latin (multus, many; plico, I fold), teaches us to express in one number what another will amount to if taken so many times. Thus, if I say multiply 29 by 17, I mean, find what 29 would amount to if taken 17 times; it will thus be seen that Multiplication is only a short way of doing certain sums in addition, because multiplying 29 by 17 is the same as putting down seventeen twenty-nines one under another and adding them together. The number to be multiplied is called the multiplicand (multiplicandum), and the number of times it is to be taken the multiplier, the result is called the product.

The factors of a number are the numbers which, being multiplied together, produce that number; as 8 and 7, factors of 56.

A prime number cannot be resolved into factors, as 17. A composite number is one that is produced by multiplying two or more simple numbers together.

When the multiplier does not exceed 12, the operation is performed in one line: you begin by multiplying the

first or extreme right hand figure of the multiplicand by the multiplier; put down the product if it is only one figure under the multiplier, if more than one figure, put down the units' figure of the product, carrying the remaining figures; then multiply the second figure of the multiplicand by the multiplier, and to the product add what you carried from the last product and put down the units' figure of the sum under the figure you multiplied, carrying the others to the next product; continue this operation till you have multiplied all the figures of the multiplicand, observing to put down the whole of the last product.

In working the example given, observe to draw a line under the multiplier for the whole length of the multiplicand; then say 5 times 5 are 25, put down 5 and carry 2; then say 5 times 2 are 10; add the 2 you carried makes 12, put down 2 and carry 1, and so on till the whole is completed.

EXAMPLE.

379625
5
1898125

EXERCISE 5.

(1)	37954638	18977319
	2	4
(2)	79621378	39815689
	3	6
(3)	79163896	39581948
	5	10
(4)	90981639	70763497
	7	9
(5)	4551216	4171948
	11	12

COMPOSITE NUMBERS.

Resolve the number into its factors and multiply by each successively; the last product will be the answer.

(6)	Multiply	29176348	by 24	(8)	Multiply	94362792	by 45
	„	14588174	by 48		„	47181346	by 90
(7)	„	79367946	by 36	(9)	„	89463898	by 54
	„	39683973	by 72		„	44731949	by 108

FRACTIONAL NUMBERS.

(10)	Multiply	97946384	by $6\frac{1}{4}$	(11)	Multiply	56179268	by $3\frac{3}{4}$
	„	48973192	by $12\frac{1}{2}$		„	28589634	by $7\frac{1}{2}$

(12) Multiply 93061728 by $9\frac{3}{4}$
„ 46530864 by $19\frac{1}{2}$

Note.—To multiply by $\frac{1}{4}$ divide by 4; by $\frac{1}{2}$ divide by 2; by $\frac{3}{4}$ multiply by 3, and divide by 4, or take half and then a half of that, and add the two results together.

When the multiplier is a prime number above 11, or any large number, multiply by each figure separately, placing each result one figure further to the left hand. The total of the products will be the answer.

(13)	Multiply	42946386	by 29	(19)	Multiply	9164536	by 3794
	„	14315462	„ 87		„	4582268	„ 7588
(14)	„	29379845	„ 78	(20)	„	8634291	„ 2573
	„	58759690	„ 39		„	2878097	„ 7719
(15)	„	89467986	„ 47	(21)	„	5427638	„ 4173
	„	44733993	„ 94		„	2713819	„ 8346
(16)	„	79146867	„ 597	(22)	„	37928164	„ 43527
	„	26382289	„ 1791		„	18964082	„ 87054
(17)	„	76193698	„ 379	(23)	„	76394658	„ 39762
	„	38096849	„ 758		„	38197329	„ 79524
(18)	„	94628910	„ 487	(24)	„	3101013	„ 1013010
	„	47314455	„ 974		„	9303039	„ 337670

When a cypher occurs among the figures of the multiplier, place the first figure of the next product line under the third figure of the preceding line ; 0 is no multiplier.

Exercise 6.

(1)	Multiply	37916	by 23015	(3)	Multiply	201362	by 70132
	„	18958	„ 46030		„	402724	„ 35066
(2)	„	101010	„ 7905	(4)	„	7317924	„ 3001014
	„	50505	„ 15810		„	14635848	„ 1500507

If the multiplicand or multiplier ends with ciphers place the same number as is contained at the end of the multiplier and multiplicand at the beginning of the first product line, and continue the operation, taking no further notice of the ciphers except to annex them to the product, as in the example annexed.

EXAMPLE.

312,0
27,00
2184000
624
8424000

(5) Multiply 31010 by 2700 (6) Multiply 58000 by 59000
„ 62020 „ 1350 „ 29000 „ 118000

(7) Multiply 31700 by 18500
„ 63400 „ 9250

EXERCISE 7.

(1) Multiply three hundred and seventy-five by four hundred and ninety-nine. Multiply two thousand four hundred and ninety-five by seventy-five.

(2) Multiply two hundred and twenty-six times three hundred and eighty-nine by five hundred and four.

Multiply one thousand and eight by one hundred and thirteen times three hundred and eighty-nine.

(3) There are twenty-nine boxes, each containing two hundred and seventy-nine apples, how many are there in all?

In each of eighty-seven boxes there are ninety-three oranges, how many are there in all?

(4) In one book there are 27963 lines of letter-printing, how many lines are there in an edition of 2956 such books?

A certain sum of money amounts to 9321 farthings, how many farthings are there in 8868 such sums?

(5) There are 63360 inches in a mile, how many inches would reach 795 miles?

A certain vessel will hold 7040 pints, how many pints would 7155 such vessels contain?

(6) There are 29946 cubic inches in a butt, how many are there in 3909 butts?

There are 14973 cubic inches in half a butt, how many are there in 7818 half butts?

DIVISION.

Division is a process by which we find how many times one number is contained in another; or, in other words,

how many times a small number can be taken from a larger. It is thus a short method of doing certain examples in subtraction. If I say divide 49 by 9, I mean, find how many nines are contained in 49, or find how many times I can take 9 from 49 till less than 9 or 0 remains. The number divided is the dividend, the other the divisor, and the result the quotient. To prove a division sum, multiply the quotient by the divisor and to the result add the remainder, if any, of the division sum, and it will give the dividend.

Rule for Short Division.

Operation.—Place the divisor to the left of the number to be divided, and in a straight line with it. Find how many times the divisor is contained in the left hand figure of the dividend. Should this figure be a lower one than the divisor, the next following must be taken with it; and then having found how many times the divisor is contained in them, place the number of times under the last figure taken of the dividend; if there be any remainder, consider it so many tens, to which add the next figure of the dividend, ascertain how many times the divisor is contained in it, and place the result under the third figure; continne this operation till the division is completed. Thus, in working the example given above, I say nines in 79; there are 8 and 7 over, put down the 8 and consider the 7 over as so many tens, to which add the third figure of the dividend, making 71; then say nines in 71, 7 and 8 over, put down the 7 and carry the 8, which treat in the same way as the last remainder, and so on to the last figure of the dividend.

EXAMPLE.
9)79149
8794-3

9
79149

The above rule applies to simple short Division, which is a process used when the Divisor is not a higher number than 12; but when it is a large number it must be worked by a Long Division process.

Rule for Long Division.

Place the divisor to the left of the dividend and in a line with it. Then see how many times the divisor is contained in the same number of figures on the left side of the dividend. If the divisor be not contained in the same number of figures in the dividend the next figure must be included, then, having ascertained the number of times the divisor is contained in the figures you have taken of the dividend, place the number of times in the quotient to the right of the dividend, multiply the divisor by this figure and place the product under the figures you have taken of the dividend, and subtract; to the remainder annex the next figure of the dividend and find what number of times the divisor is contained in it; put the number of times in the quotient to the right of the first quotient figure, multiply the divisor by the last quotient figure, and subtract the product from the remainder to which you have just annexed the dividend figure; then annex the next dividend figure to the remainder, and continue this process till you have brought down all the dividend figures.

Example to illustrate the above Rule.

Begin by ascertaining how many times 257 is contained in 916, which you find to be 3. Place 3 as the first quotient figure, multiply 257 by 3, and place the product under the figures you have taken of the dividend; subtract 771 from 916, then to the right of the remainder, 145, place 3, then find how many times 257 is contained in 1453, put the number of times in the quotient next to the last figure 3, multiply the divisor by 5, subtract the product from 1453, and proceed as before.

```
257)91635(352
    771
   1453
   1385
    685
    514
    171
```

Note.—In all division sums the remainder, if any, of the second question in each example will be a multiple of the remainder of the first question, or produce the same fraction.

Exercise 8.

(1) Divide 39618467 by 2.
„ 79236934 by 4.
(2) „ 28103625 by 3.
„ 56207250 by 6.
(3) „ 71837667 by 9.
„ 55873741 by 7.
(4) „ 37916018 by 4.
„ 75832036 by 8.
(5) „ 43989765 by 5.
„ 87979530 by 10.
(6) „ 10996244 by 12.
„ 10079894 by 11.
(7) „ 4096847360 by 6, 9 and 12 successively.
„ 4096847360 by 8, 9, and 9 successively.
(8) „ 949965640 by 2, 3, 4, and 5 successively.
„ 949965640 by 10 and 12 successively.
(9) „ 175218120 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 successively, and the same dividend by 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, successively.

COMPOSITE NUMBERS.

When the divisor is a composite number it is easier to divide by its factors, in which case there are some times as many remainders as there are divisors, and when the last quotient is obtained it is necessary to form one remainder from the other remainders; or, to find what would have been the remainder had the result been obtained by a single process of division.

To find the true remainder.

Rule.—Multiply the last remainder by the divisor of the line from which it is over, and add the preceding remainder, if any; then multiply this sum by the divisor preceding the last divisor you multiplied by, and add the remainder, if any, immediately preceding the last remainder you added, and so on till you have multiplied by the first divisor and added the first remainder, that is, what was over from the first division, and the final result is the true remainder.

As it is usual to adopt this plan of division only when the composite number can be resolved into two easy factors, the following easy rule may be committed to memory :— Multiply the first divisor by the last remainder, adding to the product the first remainder, the sum will be the true remainder.

Exercise 9.

(1) Divide 36458936 by 24 (3) „ 61032064 by 36
„ 9114734 by 96 „ 30516032 by 72
(2) „ 97181648 by 54 (4) „ 87440367 by 27
„ 43590824 by 108 „ 29146789 by 81

(5) Divide 90883911 by 48
„ 30294637 by 144.

DIVISION BY PRIME NUMBERS ABOVE 11, AND NUMBERS THAT ARE NOT EASILY RESOLVED INTO SIMPLE FACTORS.

Exercise 10.

(1) Divide 915277847 by 23 (5) Divide 10068056317 by 253
„ 1472403493 by 37 „ 30204168951 by 759
(2) „ 8237500623 by 207 (6) „ 4738625315 by 371
„ 11779227944 by 296 „ 33170377198 by 2597
(3) „ 2745833541 by 69 (7) „ 6423638973 by 3279
„ 2944806986 by 74 „ 70660028703 by 36069
(4) „ 3645378926 by 29 (8) „ 4981637256 by 5796
„ 7690757852 by 58 „ 54798098016 by 63756

When the divisor ends in one or more ciphers, cut them off, and also cut off the same number of figures from the right end of the dividend, and divide the reduced dividend by the reduced divisor; to the remainder, however, the figures cut off from the dividend must be annexed.

(9) Divide 379163486 by 230 (10) Divide 716329143 by 131000
„ 7583269720 by 4600 „ 2143987429 by 39300

(11) Divide 794006103 by 2101000
„ 2382018309 by 6303000

DIVISION BY NUMBERS CONTAINING FRACTIONS.

Multiply that part of the divisor which is not a fraction by the lower figure of the fraction and add in the top

figure; also multiply the dividend by the same number, and you will have a new dividend and divisor, with which proceed as in ordinary long division. To get the true remainder, divide the remainder you have obtained by the number you multiplied the dividend by, and the quotient will be the true remainder.

(12) Divide 79163425 by $2\frac{1}{4}$
„ 237490275 by $6\frac{1}{4}$

(13) Divide 82161035 by $4\frac{1}{2}$
„ 246483105 by $13\frac{1}{2}$

Exercise 11.

(1) I have five hundred and ninety-nine apples to divide among nine boys; how many will that be for each?

Divide one thousand one hundred and ninety-eight nuts among eighteen boys; how many will that be for each?

(2) How many boxes can be filled with 3919615 matches, putting 279 in each?

How many times is 558 contained in 7839230?

(3) Gave a sum of money equal to 37916 pence for $73\frac{1}{2}$ yards of cloth; how many pence is that per yard?

There are 75832 leaves in 147 books; how many leaves on an average is that to each book?

(4) A gentleman gave a sovereign to be divided among 48 boys; how much is that for each?

Bought 99 books for 2*l.* 1*s.* 3*d.*, how much is that for each book?

(5) Fourteen men tied 6174 trusses of hay in seven days; how many did each tie per day?

Sixteen pieces of cloth, each containing $12\frac{1}{4}$ yards, cost 12348 pence; how many pence is that per yard?

COMPOUND RULES.

Hitherto the numbers dealt with have represented simple quantities; we shall now apply the elementary rules to compound or concrete quantities, *i.e.* quantities or expressions containing several denominations.

Before being able to work these examples easy, it will be necessary to learn thoroughly the money tables and the weights and measures.

COMPOUND ADDITION.

The quantities to be added must be placed under each other so that terms of the same denomination shall be placed in the same column.

Having placed the quantities in proper order, with the names at the top of each column, and it is well for beginners to put above the name on each column the number of times the quantity represented by the name on the top of each is contained in the quantity named on the column immediately to the left of it: thus, in adding cwt. qrs. and lbs. together, place 28 over the lbs. and 4 over the qrs.

Operation.—Add up the right hand column, and if it comes to less than the number you have placed at the top of the column put it down under the first column; if it comes to more, divide it by the number at the top of the column, carry the quotient to the next column, and, putting down the remainder, if any, under the first column, continue the same process with the second column, and so on till the addition is complete. Prove by adding downwards.

EXAMPLE.

	20	12	4
£27	19		4½
33	16		6¼
57	17	11	10¾

Illustration of the Example.—Add up the farthing column and you will find it comes to 6, six farthings are 1½*d.*, put down the halfpenny and carry the 1 to the pence column; add up the pence column, and it will come to 22, twenty-two pence are 1*s.* 10*d.*, put down 10*d.* and carry the 1 to the shillings; add up the shillings, and it comes to 53, fifty-three shillings are £2 13*s.*, put down 13*s.* and carry the 2 to the pounds, then finish as in ordinary addition.

The sums in this example may be also proved by dividing each into two sums and adding the results.

Exercise 12.

(1)

£	s.	d.
37	15	4½
29	13	11½
73	15	6½
59	17	3½

£	s.	d.
48	17	5½
13	13	1½
35	16	7
62	18	8½

(2)

£	s.	d.
79	19	3½
86	17	10½
53	10	9½
26	19	4½
56	16	3½

£	s.	d.
68	15	2½
64	15	8½
64	11	11
49	1	6½
56	16	3½

(3)

£	s.	d.
39	17	11½
56	16	10½
45	19	3½
89	18	6½
73	15	10½
37	17	11½

£	s.	d.
28	16	10½
67	17	11½
56	19	4½
78	18	5½
62	14	9½
48	19	0½

(4)

£	s.	d.
562	16	4½
437	3	7½
359	19	6½
640	0	5½
795	19	6½
875	19	8½

£	s.	d.
379	14	3½
620	5	8½
684	0	5½
987	18	9½
295	16	3½
704	3	8½

(5)

£	s.	d.
395	17	2½
604	2	9½
323	10	5½
676	9	6½
298	19	10½
879	19	9½
564	16	3½

£	s.	d.
604	10	2½
395	9	9½
409	0	11½
768	8	8½
564	16	3½
298	19	6½
702	10	5½

(6)

lbs.	ozs.	dwts.	grs.
237	9	11	13
795	10	13	17
864	11	19	21
296	3	8	6
579	10	7	12

lbs.	ozs.	dwts.	grs.
459	11	13	15
795	10	13	17
642	9	17	19
387	7	6	9
488	6	9	9

(7)

cwt.	qrs.	lbs.
379	2	17
358	3	16
799	1	23
614	2	25
899	1	19

cwt.	qrs.	lbs.
490	3	18
247	2	15
921	3	25
492	0	23
899	1	19

(8)

tons.	cwt.	qrs.	lbs.	ozs.	drs.
799	16	3	17	10	11
837	19	2	16	13	12
266	17	1	13	11	17
788	16	3	27	13	15
989	13	1	18	11	10

tons.	cwt.	qrs.	lbs.	ozs.	drs.
688	15	2	16	9	10
948	0	3	17	14	13
277	18	2	14	13	18
899	17	0	0	14	0
767	10	3	16	9	8

(9)

lbs.	ozs.	drs.	scr.	grs.
791	9	7	2	17
389	6	3	1	16
519	11	6	0	13
895	10	5	1	10
486	11	3	0	8

lbs.	ozs.	drs.	scr.	grs.
902	10	4	0	18
611	8	6	0	18
408	10	5	0	12
895	10	5	1	10
264	9	5	0	6

(10)

yds.	qrs.	nls.
733	2	2
266	1	2
517	3	1
482	0	3
299	2	2
789	3	3

yds.	qrs.	nls.
844	3	3
266	1	2
406	2	0
259	2	1
400	3	3
911	1	0

(11)

hhds.(wine)	gls.	pts.
395	17	7
389	16	5
896	27	3
784	19	5
899	18	6
789	29	3

hhds.	gals.	pts.
1228	24	1
452	22	7
284	16	6
500	17	6
734	22	5
954	25	4

(12)

barls. (beer)	gls.	pts.
376	17	4
695	3	3
398	17	5
389	16	1
517	4	6

barls.	gals.	pts.
305	2	4
409	18	6
500	17	2
154	15	2
917	5	5

(13)

miles.	fur.	poles.
730	7	27
395	5	31
624	3	16
598	4	17
789	6	32

miles.	fur.	poles.
286	3	23
273	3	29
846	5	18
820	6	19
911	8	34

(14)

sq. yds.	ft.	ins.
399	7	104
625	3	98
786	6	117
298	8	109
564	4	89

sq. yds.	ft.	ins.
126	1	122
892	0	104
766	4	101
532	2	112
358	3	78

(15)

cub. yds.	ft.	ins.
793	23	1012
849	17	895
862	10	586
938	16	894
937	10	275

cub. yds.	ft.	ins.
460	20	609
516	14	562
973	11	697
1160	18	1186
1270	13	608

(16)

days	hrs.	min.	sec.
737	11	13	27
396	17	10	35
483	10	23	31
516	13	39	47
796	17	47	26

days	hrs.	min.	sec.
848	22	24	38
285	5	59	24
261	8	21	29
627	14	40	48
907	18	48	27

Exercise 13.

(1) Add together the following sums: 2*l.* 6*s.* 9¼*d.*; 13*s.* 6½*d.*; 10*d.*¾; 3*l.* 15*s.* 4¼*d.*, and 19*s.* 10*d.*

Find the amount of the following sums: 3*l.* 7*s.* 10½*d.*; 2*l.* 14*s.* 3*d.*; 17*s.* 8*d.*; 12*s.* 8½*d.*, and 3*s.* 10¾*d.*

(2) Laid out in bread, 9*s.* 10¾*d.*; meat, 15*s.* 6¾*d.*; vegetables, 14*s.* 9½*d.*; and sundries 4*s.* 6½*d.*; how much was laid out in all?

Expended the following sums: 12*s.* 1¼*d.*; 13*s.* 4¼*d.*; 10*s.* 5½*d.*; and 8*s.* 10½*d.*; how much was expended in all?

(3) Seven post offices sold stamps in one year to the following amounts: 793*l.* 19*s.* 4*d.*; 862*l.* 10*s.* 11*d.*; 365*l.* 11*s.* 4*d.*; 1013*l.* 11*s.* 9*d.*; 237*l.* 19*s.* 6*d.*; 143*l.* 14*s.* 8*d.*, and 99*l.* 19*s.* 1*d.*; how much did it all amount to?

Shipped goods to the following amounts: 682*l.* 18*s.* 3*d.*; 751*l.* 9*s.* 10*d.*; 587*l.* 13*s.* 6*d.*; 680*l.* 8*s.* 6*d.*; 149*l.* 0*s.* 7*d.*; 254*l.* 15*s.* 9*d.*; 411*l.* 0*s.* 2*d.*; what was their total value?

(4) A gentleman's silver spoons weigh 3 lbs. 11 oz. 17 dwts. 16 grs.; his forks, 4 lbs. 10 oz. 19 dwts. 23 grs.; his silver tea service, 5 lbs. 10 oz. 13 dwts. 14 grs.; and a large tankard, 10 lbs. 10 oz. 11 dwt. 11 grs.; what weight is there in all?

The following quantities of gold were assayed at one office: 5 lbs. 13 oz. 19 dwts. 18 grs.; 2 lbs. 8 oz. 17 dwts. 21 grs.; 12 lbs. 4 oz. 19 dwts. 20 grs.; and 4 lbs. 4 oz. 5 dwts. 5 grs.; how much was assayed in all?

(5) A soap merchant's monthly business was as follows:—1st week, 375 cwt. 2 qrs. 10 lbs.; 2nd week, 299 cwt. 3 qrs. 17 lbs.; 3rd week, 473 cwt. 1 qr. 10 lbs.; and 4th week, 317 cwt. 2 qrs. 5 lbs.; what was the total?

In four weeks a grocer sold the following quantities of sugar: 584 cwt. 2 qrs. 11 lbs.; 206 cwt. 1 qr. 4 lbs.; 264 cwt. 1 qr. 9 lbs., and 411 cwt. 18 lbs.; what is the total?

(6) A farmer obtains the following crops of corn in one year: Wheat, 397 qrs. 3 bus. 3 pks.; oats, 52 qrs. 1 bus. 2 pks.; barley, 137 qrs. 5 bus. 1 pk.; beans, 89 qrs. 4 bus. 2 pks.; how much had he in all?

A corn dealer sold in one year the following quantities of corn: Beans, 167 qrs. 4 bus. 1 pk.; wheat, 319 qrs. 3 bus.; oats, 63 qrs. 3 bus. 3 pks., and barley, 126 qrs. 4 bus.; what was the total sale of corn?

COMPOUND SUBTRACTION.

In subtracting compound quantities, place the same denominations under each other, and proceed as in ordinary subtraction, observing when the upper quantities are smaller than the lower, to add as many to the upper as make one of the next higher denomination.

In working this example, say, a halfpenny, that is 2 from

1 cannot be taken, borrow 4, because 4 in that column make 1 in the next; 4 and 1 are 5, 2 from 5 remain 3, put down $\frac{3}{4}d.$ and carry 1, because 4 were borrowed; 1 and 4 are 5, 5 from 8 remain 3, put down 3; 17 from 11 cannot be taken, add 20, 17 from 31 leaves 14, carry 1 to the unit's figure in the £'s and then proceed as in simple subtraction.

EXAMPLE.

£	s.	d.
379	11	$8\frac{1}{4}$
136	17	$4\frac{3}{4}$

In the following examples, prove the first operation by the double subtraction in the second part of the example.

EXERCISE 14.

(1)

£	s.	d.
793	16	4
231	11	3

£	s.	d.
904	17	5
228	14	10
...	..	..
113	17	6
...	..	..

(2)

£	s.	d.
1010	10	$1\frac{1}{2}$
101	11	$2\frac{3}{4}$

£	s.	d.
2121	11	$2\frac{3}{4}$
918	14	$7\frac{1}{2}$
...	..	..
293	17	$8\frac{1}{2}$
...	..	..

(3)

£	s.	d.
796	13	$3\frac{3}{4}$
235	18	$9\frac{1}{2}$

£	s.	d.
685	12	$2\frac{1}{2}$
87	1	$3\frac{3}{4}$
...	..	..
37	16	$4\frac{1}{2}$
...	..	..

(4)

£	s.	d.
936	10	$3\frac{1}{2}$
289	17	$11\frac{1}{4}$
..	..	..

£	s.	d.
825	9	$2\frac{1}{4}$
148	16	11
...	..	..
29	19	11

(5)

£	s.	d.
3725	19	$11\frac{1}{2}$
1367	19	$11\frac{3}{4}$

£	s.	d.
2613	18	$10\frac{1}{2}$
196	0	$10\frac{3}{4}$
...	..	..
59	17	$11\frac{3}{4}$
...	..	..

(6)

lbs.	ozs.	dwts.	grs.
79	11	17	19
23	10	18	22
..	..	..	..

lbs.	ozs.	dwts.	grs.
91	0	18	20
11	1	2	4
..	..	..	..
23	10	17	19

(7)

lbs.	ozs.	drs.	scr.	grs.
93	11	3	1	17
27	10	5	9	18

lbs.	ozs.	drs.	scr.	grs.
104	0	4	2	18
23	10	3	2	16
..	..	..	..	..
14	1	3	1	3

(8)

tons	cwt.	qrs.	lbs.
937	17	2	21
216	17	3	24

tons	cwt.	qrs.	lbs.
815	15	0	19
66	17	2	27
..	..	..	..
27	17	2	23
..	.	..	..

(9)

yds.	qrs.	nls.
953	3	2
237	3	3

yds.	qrs.	nls.
842	2	1
89	3	1
..	..	..
36	3	1
..	..	..

(10)

hhds.(wine)	gals.	pts.
799	33	7
226	49	3

hhds.(wine)	gals.	pts.
688	22	6
36	12	7
..	..	..
79	25	3

(11)

kil.	gals.	pts
453	13	3
129	17	2

kil.	gals.	pts.
675	15	5
222	5	5
..	..	..
129	13	7

(12)

qrs.	bus.	pks.
701	4	1
136	5	2

qrs.	bus.	pks.
812	5	2
157	7	1
..	..	..
89	7	2
..	..	..

(13)

miles	fur.	poles.
800	1	27
139	2	33

miles	fur.	poles.
833	4	30
96	7	7
..	..	..
75	6	29

(14)

sq. yds.	ft.	ins.
373	7	101
189	7	123

sq. yds.	ft.	ins.
406	2	134
142	4	131
..	..	..
79	7	25

(15)

cub. yds.	ft.	ins.
395	19	1013
136	22	1325

cub. yds.	ft.	ins.
728	22	1346
256	6	863
..	..	..
213	19	795
..	..	..

(16)

days	hrs.	min.	sec.
796	17	18	23
235	23	19	57

days	hrs.	min.	sec.
885	6	7	12
187	2	33	19
..	..	..	..
137	9	35	27
..	..	..	..

Exercise 15.

(1) What sum of money added to 799*l.* 16*s.* 11¼*d.* will make the sum 3796*l.* 17*s.* 9¾*d.*?

Paid away 910*l.* 17*s.* 0½*d.* out of 3907*l.* 18*s.* 10*d.*; how much remains?

(2) Spent the following sums out of a 5*l.* note; 6*s.* 10½*d.*; 5*s.* 11¼*d.*; 13*s.* 9¾*d.*; 7½*d.*; 1*l.* 1*s.* 2½*d.*; half-a-crown, and 4¼*d.*; how much is left?

Went to market with 7*l.* 11*s.* 3¾*d.* and spent the following sums:—13*s.* 9*d.*; 11*s.* 10½*d*; 1*l.* 7*s.* 7½*d*,; 1*s.* 3*d.*; 8½*d.*; 5*s.*, and 2*l.* 2*s.* 5*d.*; how much have I left?

(3) Take 33½ sovereigns from 50 guineas; also 138½ sovereigns from 150 guineas.

(4) Tom spent three half-sovereigns, nine half-crowns and 11*d.*, and

Harry spent 5 crowns, 3 half-crowns, 7 florins, 3 sixpences, and 2 farthings: which spent the most, and how much?

Received £1 12s. 1½d., and paid away the following sums: 1l. 3s. 1d.; and half-a-sovereign: 14 shillings, 2s. 5d., and 3s. 1¾d.; how much remains?

5. Sold 13 cwt. 1 qr. 17 lbs. out of a hogshead of sugar weighing 17 cwt. 1 qr. 4 lbs.: how much is left?

Bought 2 tons 13 cwt. 2 qrs. 12 lbs. of soap, and sold at different times 2 tons 9 cwt. 2 qrs. 25 lbs.; how much remains unsold?

6. One farm contains 396 a. 3 r. 27 p., another 157 a. 1 r. 34 p.; how much is one larger than the other?

On a farm there are 441 a. 3 r. 31 p. of pasture, and 202 a. 1 r. 88 p. of arable; what is the difference?

COMPOUND MULTIPLICATION.

Rule.—**Write the multiplier under the lowest denomination of the multiplicand, then find the product of these two numbers and divide it by the number of units of this denomination, which is equivalent to one of the next higher in the multiplicand; put the remainder, if any, under the multiplier, and carry the quotient to the product of the multiplier, and the next quantity in the multiplicand. Divide this product as you did the last, and so continue till the whole operation is complete.**

The above rule applies to multipliers not exceeding 12; when the multiplier is a higher number and composite, multiply by the factors successively; if the number be not composite, or not easily resolvable into factors, the operation is performed in a different manner which will be explained further on.

In the example given, begin thus: 3 times 3 are 9, 9 farthings are 2¼*d.*, put down ¼*d.* and carry 2; then 3 times 6 are 18, adding the 2 carried makes 20, 20 pence are 1*s.* 8*d.*, put down 8*d.* and carry 1; then 3 times 17 are 51, adding 1 makes 52, 52 shillings are £2 12*s.*, put down 12 and carry 2; 3 times 9 are 27, adding 2 makes 29, put down 9 and carry 2; 3 times 2 are 6 and 2 carried make 8.

EXAMPLE.

£	s.	d.
20	17	6¾
		3
80	12	8¼

Exercise 16.

(1)

£	s.	d.
796	13	$6\frac{1}{2}$
		2

£	s.	d.
398	6	$9\frac{1}{4}$
		4

(2)

£	s.	d.
599	17	$11\frac{1}{2}$
		3

£	s.	d.
299	18	$11\frac{3}{4}$
		6

(3)

£	s.	d.
936	11	$1\frac{1}{2}$
		5

£	s.	d.
468	5	$6\frac{3}{4}$
		10

(4)

£	s.	d.
799	10	$5\frac{1}{2}$
		4

£	s.	d.
399	15	$2\frac{3}{4}$
		8

(5)

£	s.	d.
881	3	$10\frac{1}{2}$
		7

£	s.	d,
685	7	$5\frac{1}{2}$
		9

(6)

£	s.	d.
866	14	$7\frac{1}{2}$
		12

£	s.	d.
945	10	6
		11

Exercise 17.

COMPOSITE NUMBERS.

Multiply successively by the factors; the last product ill be the answer.

		£	s.	d.			£	s.	d.		
)	Multiply	47	19	$6\frac{1}{2}$	by 24	(3) Multiply	105	8	$10\frac{1}{2}$	by	54
	„	23	19	$9\frac{1}{4}$	„ 48	„	52	14	$5\frac{1}{4}$	„	108
)	„	59	10	$10\frac{1}{2}$	„ 36	(4) „	99	19	$11\frac{1}{2}$	„	72
	„	29	15	$5\frac{1}{4}$	„ 72	„	49	19	$11\frac{3}{4}$	„	144

When the multiplier is not above 12, and is not a composite number, find the largest composite number which is less than the given multiplier; then multiply the quantity given by the factors of this composite number, and to this product add the product of the quantity given and the difference between the composite number and the given multiplier: the result will be the product sought.

For example, in multiplying a quantity by 57, I find that 56 is the nearest composite number. Multiply by its factors, 8 and 7; there is then a difference of 1, so I multiply the top line by 1, and add it to the last result, and it will give the answer.

Exercise 18.

		£	s.	d.			£	s.	d.	
(1)	Multiply	37	19	$5\frac{1}{2}$	by 37	(4) Multiply	295	16	$11\frac{1}{2}$	by 53
	„	18	19	$8\frac{3}{4}$	„ 74	„	147	18	$5\frac{3}{4}$	„ 106
(2)	„	57	16	$2\frac{1}{2}$	„ 47	(5) „	307	10	$9\frac{1}{2}$	„ 59
	„	28	18	$1\frac{1}{4}$	„ 94	„	153	15	$4\frac{3}{4}$	„ 118
(3)	„	87	9	$4\frac{1}{2}$	„ 67	(6) „	499	16	$7\frac{1}{2}$	„ 74
	„	43	19	$8\frac{1}{4}$	„ 134	„	249	18	$3\frac{3}{4}$	„ 148

When it is not easy to find the largest composite number in a given number, it is more convenient, if it consists of hundreds, to multiply by the component parts of 100 (10 and 10), and then by the number of hundreds multiply the line which is ten times the given quantity by the tens' figure of the given multiplier, and the top line by the units' figure of the multiplier.

General rule for multiplying by large numbers.

Multiply by as many tens as there are figures in the number, less one; and the last result by the first figure, that is the figure on the extreme left of the given number; place the tens' figure of the given number to the right of the tens' product, the hundreds' figure of the given multiplier opposite the hundreds' product, and so on, placing each following figure of the multiplier opposite each following product. Having so put down all the figures of the multiplier, proceed to multiply the last product which has a multiplier

opposite to it by that multiplier, and place the product under the last product already calculated; and, in like manner, multiply each line in the sum by the number standing opposite to it, placing the results under each other, and the sum of these products will give the answer.

Examples to illustrate above Rule.

Multiply £29 16*s.* 11½*d.* by 345

£	*s.*	*d.*	
29	16	11½	× 5
		10	
298	9	7	× 4
		10	
2984	15	10	
		3	
8954	7	6	= 300
1193	18	4	= 40
149	4	9½	5
10297	10	7½	= 345

Multiply £29 17*s.* 11½*d.* by 29645

£	*s.*	*d.*	
29	17	11½	× 5
		10	
298	19	4½	× 4
		0	
2989	13	9	× 6
		10	
29896	17	6	× 9
		10	
298968	15	0	
		2	
597937	10	0	= 20000
269071	17	6	= 9000
17938	2	6	= 600
1195	17	6	= 40
149	9	8¼	5
£886292	17	2¼	= 29645

Exercise 19.

		£	*s.*	*d.*			£	*s.*	*d.*	
(1)	Multiply	79	16	8½	by 196	(6)	Multiply	7	19	6½	by 3728
	„	39	18	4½	„ 392		„	3	19	9½	„ 7456
(2)	„	83	12	6½	„ 277	(7)	„	15	11	7½	„ 4239
	„	41	16	3½	„ 554		„	7	15	9½	„ 8478
(3)	„	79	15	11½	„ 369	(8)	„	8	3	4	„ 9375
	„	39	17	11½	„ 738		„	4	1	8	„ 18750
(4)	„	9	18	10½	„ 987	(9)	„	7	11	11½	„ 29764
	„	4	19	5½	„ 1974		„	3	15	11½	„ 59528
(5)	„	8	17	6½	„ 2364	(10)	„	9	19	8½	„ 97643
	„	4	8	9½	„ 4728		„	4	19	10½	„ 195286

FRACTIONAL QUANTITIES.

Exercise 20.

	£	s.	d.			£	s.	d.	
(1) Multiply	379	10	8	by $2\frac{3}{4}$	(2) Multiply	762	11	10	by
„	189	15	4	„ $5\frac{1}{2}$	„	2287	15	6	„

		£	s.	d.		
(3)	Multiply	395	19	10	by	$8\frac{3}{4}$
	„	197	19	11	„	$17\frac{1}{2}$

WEIGHTS AND MEASURES.

Exercise 21.

(1)	Multiply	17 lbs. 9 oz. 10 dwts. 9 grs.	by	7
	„	5 lbs. 11 oz. 3 dwts. 11 grs.	„	21
(2)	„	17 cwt. 3 qrs. 17 lbs.	„	9
	„	3 cwt. 2 qrs. 9 lbs.	„	45
(3)	„	55 tons 15 cwt. 1 qr. 14 lbs.	„	12
	„	7 tons 19 cwt. 1 qr. 10 lbs.	„	84
(4)	„	166 lbs. 13 oz. 6 drs.	„	11
	„	16 lbs. 10 oz. 15 drs.	„	110
(5)	„	120 qrs. 7 bu. 2 pks.	„	19
	„	13 qrs. 3 bu. 2 pks.	„	171
(6)	„	38 miles 1 fur. 13 poles	„	23
	„	4 miles 1 fur. 37 poles	„	207
(7)	„	15 sq. yds. 7 ft. 36 in.	„	59
	„	1 sq. yd. 6 ft. 116 in.	„	531
(8)	„	209 cub. yds. 23 ft. 1092 in.	„	95
	„	17 cub. yds. 13 ft. 379 in.	„	1140
(9)	„	124 days 2 hrs. 30 min.	„	27
	„	17 days 17 hrs. 30 min.	„	189
(10)	„	81 hrs. 47 min. 4 sec.	„	52
	„	10 hrs. 13 min. 23 sec.	„	416
(11)	„	58 acres 2 rds. 10 poles	„	100
	„	5 acres 3 rds. 17 poles	„	1000
(12)	„	36 cwt. 1 qr. 10 lbs.	„	255
	„	3 cwt. 2 qrs. 15 lbs.	„	2550

Exercise 22.

			£	s.	d.	
(1)	27 yards of cloth,	@	0	3	$9\frac{1}{2}$	per yard
	54 „ „	„	0	1	$10\frac{3}{4}$	„

			£	s.	d.	
(2)	59 books,	@	0	7	6	each.
	118 „	„	0	3	9	„
(3)	79 cwt. of sugar,	„	2	16	11½	per cwt.
	158 „ „	„	1	8	5¾	„ „
(4)	217 yards of ribbon,	„	0	0	7½	per yard.
	434 „ „	„	0	0	3¾	„ „
(5)	863 oz. of silver,	„	0	5	8½	per oz.
	1726 „ „	„	0	2	10¼	„ „
(6)	989 ozs. of gold,	„	3	18	10½	„ „
	1978 „ „	„	1	19	5¼	„ „
(7)	2389 yards of calico,	„	0	0	9½	per yard.
	4778 „ „	„	0	0	4¾	„ „

EXERCISE 23.

(1) What is the value of 29 yds. of velvet, at the rate of 17*s.* 6½*d.* per yd.?

At the rate of 8*s.* 9¼*d.* per cwt., what is the value of 58 cwt. of soda?

(2) What is the value of 27 pieces of cloth, each containing 28 yds., at 2*s.* 4½*d.* per yd.?

What is the value of 18 packages of books, each containing 7 dozen, at 1*s.* 2¼*d.* each book?

(3) Required the value of 45 loads of hay, at 2*s.* 4½*d.* per truss.

Find the value of 90 loads of straw, at 1*s.* 2¼*d.* per truss.

(4) A gang of 98 men are employed on a job 9 weeks, the average daily pay of each being 3*s.* 4*d.* and 2½*d.* per day for beer; how much was paid to them in all?

What is the value of 168 pieces of cloth, each containing 63 yds., at the rate of 1*s.* 9½*d.* per yard?

(5) If you save 1½*d.* a day, how much will you save in three years?

What is the worth of 2190 yds. of paper, at 0¾*d.* per yard?

(6) Required the weight of 49 packages of sugar, each 3 lbs. 10 oz.?

Sold in one day 6 lbs. 2 oz. of tobacco, how much should I sell in 29 days at that rate?

(7) I have an order for 279 coats, and each requires 2 yds. 3 qrs. 3 nails of cloth; how much stuff shall I want for all? Also, multiply 17 yds. 1 qr. 3 nls. by 47.

(8) A barrel leaks at the rate of 1 qt. 1 pt. a day, how much will it leak in 29 days?

A man drinks $1\frac{1}{2}$ pt. of beer a day, how much will he have drunk in 58 days?

(9) The distance between two turnpike gates is exactly 6 miles 5 fur. 15 poles 3 yards; what distance must a man walk who goes over the ground 157 times?

In one journey an omnibus goes over 3 miles 2 fur. 27 poles $4\frac{1}{4}$ yds., to what distance would 314 such journeys be equivalent?

COMPOUND DIVISION.

In dividing a compound quantity by an abstract number, divide the highest denomination by the given divisor, and put the quotient under the dividend; if there is any remainder, multiply it by the number of units of the second denomination, which make one of the first, adding thereto the number, if any, of the second denomination; divide this sum by the divisor, putting the quotient, as before, under the dividend, and carrying the remainder to the next lower denomination; and so on till the whole line has been divided. Prove by multiplying the quotient by the divisor.

In working the annexed example, begin by dividing 7 by 2, put down 3 and carry 1; 2 in 19, 9 times, put down 9 and carry 1; this 1 over is £1, or 20 shillings, to which add the 16 shillings in the dividend, making 36, then say, 2 in 36, 18 times exactly; then 2 in 4 (pence), 2, put down 2; 2 in $\frac{1}{2}d.$ 1, or $\frac{1}{4}d.$, put down $\frac{1}{4}d.$, and the divison is complete.

EXAMPLE.

£	s.	d.
2)79	16	$4\frac{1}{2}$
39	18	$2\frac{1}{4}$
	2	
79	16	$4\frac{1}{2}$

EXERCISE 24.

(1)

£	s.	d.
2)793	19	$6\frac{1}{2}$

£	s.	d.
4)1587	19	1

(2)

3)804	18	$11\frac{1}{2}$

6)1609	17	11

(3)

£	s.	d.
5)374	16	3½

£	s.	d.
10)749	12	7

(4)

£	s.	d.
4)399	19	9¼

£	s.	d.
8)799	19	6½

(5)

£	s.	d.
7)5531	19	9¾

£	s.	d.
9)7112	11	2¼

(6)

£	s.	d.
11)2617	14	3½

£	s.	d.
12)2855	13	9

Exercise 25.

		£	s.	d.		
(1)	Divide	4798	8	9	by 3, 4 and 5, successively.	
	„	9596	17	6	„ 12, and 10	„
(2)	„	14395	6	3	„ 4, 5, and 9	„
	„	28790	12	6	„ 6, 6, and 10	„
(3)	„	2399	4	4½	„ 2, 3, and 5	„
	„	21592	19	4½	„ 5, 6, and 9	„

DIVISION BY COMPOSITE NUMBERS.

Divide by the factors successively, and find the true remainder as in simple division.

		£	s.	d.			£	s.	d.	
(4)	Divide	7916	10	4½	by 24	(7) Divide	5199	13	6¾	by 45
	„	15833	0	9	„ 48	„	10399	7	1½	„ 90
(5)	„	3257	19	9¾	„ 36	(8) „	4327	11	8¼	„ 72
	„	6515	19	7½	„ 72	„	8655	3	4½	„ 144
(6)	„	4738	10	11¼	„ 54					
	„	9477	1	10½	„ 108					

FRACTIONAL NUMBERS.

Multiply the dividend by the lower figure of the fraction for a new dividend, and multiply that part of the divisor which is not a fraction by the same number, adding in the top figure of the fraction for a new divisor. When this divisor does not exceed 12, proceed as in a former case:

for divisors above 12, proceed by the long division process, reducing the remainders as in short division.

Exercise 25 *a*.

		£	s.	d.			£	s.	d.		
(1)	Divide	372	16	3¼	by	3¼	727	16	4¼	by	2¾
	„	1118	8	9¾	„	9¾	2183	19	0¾	„	8¼
(2)	„	792	15	2½	„	5½	597	17	11½	„	5¼
	„	2378	5	7½	„	16½	3587	7	9	„	31½

Division by prime numbers and by numbers not easily resolved into factors.

		£	s.	d.		
(5)	Divide	3796	17	4¼	by	29
	„	7593	14	8½	„	58
(6)	„	5796	15	9¾	„	37
	„	11593	11	7½	„	74
(7)	„	8794	19	11½	„	59
	„	26384	19	10½	„	177
(8)	„	29376	15	10¼	„	87
	„	117507	3	5	„	348
(9)	„	71936	11	0¾	„	297
	„	143873	2	1½	„	594
(10)	„	839706	16	4¼	„	797
	„	2519120	9	0¾	„	2391
(11)	„	970163	18	11½	„	3927
	„	1940327	17	11	„	7854
(12)	„	781362	14	9¾	„	79
	„	9594990	2	11¼	„	869
(13)	„	594657	15	11½	„	3501
	„	1189315	11	11	„	7002
(14)	„	710816	18	4¼	„	3724
	„	3554084	11	9¼	„	18620

Let it be remarked that the remainder of the second sum of each set will be always a multiple of the remainder of the first.

Dividing by numbers ending in ciphers.

Cut off the ciphers from the divisor, and the same num-

ber of figures from the number of the first denomination, and divide as before, remembering to annex to each remainder the figures cut off from the last denomination before multiplying it. Then cut off from the new dividend the same number of figures as before, and so proceed till the operation is completed.

		£	s.	d.		
(15)	Divide	3796	17	10½	by	100
	„	37968	18	9	„	1000
(16)	„	5729	16	4¼	„	100
	„	572891	15	5	„	10000
(17)	„	57289	3	6½	„	1000
	„	5729817	14	2	„	100000

WEIGHTS AND MEASURES.

EXERCISE 26.

(1)	Divide	79 cwt. 3 qrs. 10 lbs.	by	4
	„	159 cwt. 2 qrs. 20 lbs.	„	8
(2)	„	59 lbs. 11 ozs. 10 dwts. 13 grs.	„	5
	„	119 lbs. 11 oz. 1 dwt. 2 grs.	„	10
(3)	„	73 yds. 3 qrs. 2 nls.	„	6
	„	147 yds. 3 qrs.	„	12
(4)	„	379 qrs. 7 bus. 3 pks.	„	3
	„	759 qrs. 7 bus. 2 pks.	„	6
(5)	„	379 dys. 10 hrs. 15 min.	„	24
	„	758 dys. 21 hrs. 30 min.	„	48
(6)	„	573 sq. yds. 7 ft. 100 in.	„	36
	„	1147 sq. yds. 6 ft. 56 in.	„	72
(7)	„	827 cub. yds. 21 ft. 1000 in.	„	54
	„	1655 cub. yds. 16 ft. 272 in.	„	108
(8)	„	59 tons 17 cwt. 3 qrs. 10 lbs.	„	3¾
	„	598 tons 18 cwt. 1 qr. 16 lbs.	„	37½
(9)	„	7329 dys. 17 hrs. 16 min. 53 sec.	„	4¼
	„	36648 dys. 14 hrs. 24 min. 25 sec.	„	21¼
(10)	„	813 lbs. 11 oz. 10 dwts. 17 grs.	„	89
	„	1627 lbs. 11 oz. 1 dwt. 10 grs.	„	178
(11)	„	799 tons 17 cwt. 3 qrs. 15 lbs.	„	259
	„	1599 tons 15 cwt. 3 qrs. 2 lbs.	„	518

(12)	Divide	536 yds. 3 qrs. 3 nls.	by 489
	„	1610 yds. 3 qrs. 1 nl.	„ 1467
(13)	„	9876 gals. 1 qt. 1 pt.	„ 273
	„	19753 gals. 1 qt.	„ 546
(14)	„	79896 qrs. 7 bus. 3 pks.	„ 597
	„	239690 qrs. 7 bus. 1 pk.	„ 1771
(15)	„	392618 cub. yds. 26 ft. 1000 in.	„ 9789
	„	785237 cub. yds. 26 ft. 272 in.	„ 19578

Exercise 27.

(1) Divide £274 6*s.* 6*d.* equally among 8 people.

Paid £857 5*s.* 3¾*d.* for 25 acres of ground; how much is that per acre?

(2) If 27 lbs. of beef cost a guinea, what is the price of 1 lb.?

Gave 28 shillings for 36 books; how much is that each on an average?

(3) The pay of 49 men for one week was £56 7*s.*; how much is that each per day?

There are 21 boxes, and in each box 3½ dozen of books, which altogether cost £169 1*s.*; how much is that per book?

(4) If 49 loads of hay cost £150 13*s.* 6*d.*, what is that per truss?

Gave £301 7*s.* for 294 dozens of pairs of slippers; how much is that per pair?

(5) If the pay of 279 sailors for six months' service comes to £2376 17*s.* 6*d.*, what is that each per week?

If 139½ pieces of cloth, each containing 8 dozen yards, cost £4753 15*s.*, what is that per yard?

(6) If a compositor receives £30 9*s.* for 29 weeks' work, how much is that per day?

Fourteen dozen and a half pints of wine cost as much as a gross of bottles of brandy, at £2 10*s.* 9*d.* per dozen; what is the wine worth a pint?

(7) A bullock weighing 86½ stone sells for £25 10*s.*; how much is that per lb.?

A man gets £12 15*s.* for 346 days' work; how much is that per day?

(8) How much a day at £593 10*s.* 6*d.* per year?

Paid £4154 13*s.* 6*d.* for 2555 yards of cloth; how much is that per yard?

(9) Make 49 packets of equal weight out of 4 cwt. 3 qrs. 10 lbs. of sugar ; how much in each packet ?

If 147 bags of sugar weigh 14 cwt. 2 qrs. 2 lbs., how much is that per bag ?

BILLS.

Exercise 28.

(1)

Cheesemongers' Bills.

10	lbs. butter	@	11½*d.* per lb.
21	„ cheese	„	6¾*d.* „
15	„ do.	„	9¾*d.* „
50	eggs	„	9*d.* per dozen.
13	lbs. bacon	„	8¾*d.* per lb.
25	„ lard	„	7½*d.* „
22½	„ cheese	@	6½*d.* „
31½	„ do.	„	4½*d.* „
20	„ butter	„	5¾*d.* „
19½	„ bacon	„	6½*d.* „
37½	„ do.	„	6*d.* „
100	eggs	„	7½*d.* per score.

(2)

Hosiers' Bills.

Winchester, 29th May, 1864.

Mr. Jones,

Bought of James Clark, Hosier, &c.

9	pairs stockings	@	1*s.*	4½*d.* per pair.
7	„ socks	„	0*s.*	10*d.* „
8	„ gloves	„	2*s.*	9*d.* „
6	„ do.	„	1*s.*	5*d.* „
8	„ neckties	„	2*s.*	3*d.* „
10	shirts	„	3*s.*	6*d.* „
17	yards flannel	@	1*s.*	2½*d.* per yard.
5½	„ do.	„	1*s.*	4*d.* „
7	pairs gloves	„	2*s.*	3*d.* per pair.
12	yards calico	„	0*s.*	6*d.* „ yard.
7	„ linen	„	1*s.*	11½*d.* „ „
15	shirts	„	2*s.*	4*d.* each.

(3)

Grocers' Bills.

17	cwt.	sugar	@	59*s.*	0*d.*	per cwt.
18	„	rice	„	24*s.*	0*d.*	„
13	„	tea	„	215*s.*	6*d.*	„
23	„	raisins	„	35*s.*	0*d.*	„
7	„	spice	„	5*s.*	9*d.*	per lb.
17	„	currants	„	35*s.*	0*d.*	per cwt.
23	„	sugar	@	47*s.*	6*d.*	„
19	„	rice	„	27*s.*	3*d.*	„
10	„	tea	„	198*s.*	9*d.*	„
31	„	raisins	„	27*s.*	9*d.*	„
18	„	currants	„	37*s.*	9*d.*	„
23	„	spice	„	6*s.*	7*d.*	per lb.

Sundries, £18 17*s.* 10*d.*

(4)

Vintners' Bills.

18	gals.	brandy	@	35*s.*	6*d.*	per gal.
29	„	gin	„	16*s.*	4*d.*	„
37	„	rum	„	29*s.*	3*d.*	„
17	„	whisky	„	19*s.*	7*d.*	„
15	„	spruce	„	13*s.*	9*d.*	„
19	„	peppermint	„	14*s.*	6*d.*	„
9	dozen	port	@	71*s.*	0*d.*	per dozen.
14½	„	sherry	„	32*s.*	8*d.*	„
18½	„	marsala	„	58*s.*	6*d.*	„
8½		claret	„	39*s.*	2*d.*	„
7½		Bordeaux	„	27*s.*	6*d.*	„
9½		sherry	„	29*s.*	0*d.*	„

(5)

Corn Merchants' Bills.

7	qrs.	barley	@	45*s.*	9*d.*	per. qr.
23	„	oats	„	21*s.*	6*d.*	„
15	„	peas	„	32*s.*	0*d.*	„
19	„	beans	„	25*s.*	9*d.*	„
13	„	wheat	„	49*s.*	6*d.*	„
16	„	seed	„	35*s.*	9*d.*	„

13	qrs. barley	@	38s.	6d.	per qr.
19	„ oats	„	24s.	9d.	„
17	„ peas	„	32s.	0d.	„
27	„ beans	„	17s.	6d.	„
18	„ wheat	„	53s.	6d.	„
	Sundries,	£2	9s.	3d.	

(6)

Stationers' Bills.

23	quires note paper	@	0s.	3d.	per quire.
37	„ „	„	0s.	4½d.	„
35	„ foolscap	„	1s.	3d.	„
9	reams common note paper	„	0s.	2d.	„
1000	quill pens	„	1s.	2d.	per 100.
12	gross pens	„	3s.	4d.	per gross.
34½	quires note paper	@	0s.	2d.	per quire.
55½	„ „	„	0s.	3d.	„
52½	„ foolscap	„	0s.	10d.	„
4½	reams common	„	0s.	4d.	„
500	quills	„	2s.	4d.	per 100.
24	gross pens	„	1s.	8d.	per gross.

(7)

Cloth Merchants' Bills.

20	pieces, each	27	yards,	@	13s.	3d. per yard.
10	„	19	„	„	16s.	8d. „
15	„	30	„	„	9s.	4½d. „
16	„	16	„	„	8s.	8d. „
37	„	50	„	„	2s.	3d. „
9	„	25	„	„	10s.	6d. „
18	„	25	„	@	5s.	3d. „
18½	„	50	„	„	4s.	6d. „
8	„	16	„	„	17s.	4d. „
7½	„	30	„	„	18s.	9d. „
20	„	19	„	„	8s.	4d. „
30	„	27	„	„	8s.	10d. „

(8)

Mercers' Bills.

27	yards	satin	@	5s. 10d.	per yard.
35	„	velvet	„	16s. 11d.	„
29	„	silk	„	3s. 9d.	„
51	„	superior do.	„	7s. 10d.	„
17	„	sarcenet	„	5s. 4d.	„
39	„	„	„	1s. 8d.	„
13½	„	silk	„	11s. 8d.	„
70	„	„	„	8s. 5½d.	„
43¼	„	satin	„	2s. 6d.	„
15	„	„	„	3s. 4d.	„
25½	„	velvet	„	15s. 8d.	„
34	„	„	„	2s. 8d.	„

(9)

Butchers' Bills.

175	lbs.	mutton	@	4s. 6d.	per stone.
153	„	beef	„	3s. 6d.	„
216	„	superior do.	„	5s. 0d.	„
89	„	veal	„	6s. 4d.	„
116	„	pork	„	5s. 0d.	„
43	„	suet	„	0s. 10½d.	per lb.
262½	„	mutton	„	3s. 0d.	per stone.
76½	„	beef	„	7s. 0d.	„
324	„	„	„	3s. 4d.	„
178	„	pork	„	3s. 2d.	„
174	„	„	„	3s. 4d.	„
64½	„	lamb	„	4s. 8d.	„

REDUCTION.

Reduction is a process by which we convert a simp or compound quantity to an equivalent quantity of anoth denomination.

To reduce a quantity expressed in a great denominatio

as pounds sterling, tons, &c., to an equivalent quantity of a smaller name, as farthings, drams, &c., multiply the quantity of the highest denomination given by the number of units of the next lower which equals one of the quantity multiplied; and this product by the number of units of the next denomination, which makes one of the number multiplied; and so on, till you have reduced the highest term to the one required.

To bring a quantity of a small denomination to an equivalent quantity of a large denomination, as farthings to guineas, or lbs. to tons, divide the quantity given by as many units of that quantity as make one of the next higher; and so on, till you have reduced the given quantity to the denomination sought.

When a lower denomination is not contained exactly in a higher, as pounds in guineas, bring both to a similar denomination by multiplication, and then divide the greater by the less; thus, to find how many quarter-guineas there are in so many pounds, bring the pounds to pence, and a quarter-guinea to pence, and then divide the greater by the less.

Exercise 29.

(1) Bring £47 to pence, and £94 to twopences.

(2) Bring £59 18*s.* to shillings, and £29 19*s.* to sixpences.

(3) In £793 16s. 11*d.* how many pence? and in £396 18*s.* 5½*d.* how many halfpence?

(4) In 31016 pence how many £'s? and in 15508 pence how many half-sovereigns?

(5) In 519376 pence how many guineas? and in 259688 pence how many half-guineas?

(6) Bring £7946 16*s.* 11½*d.* to farthings, and £15893 13*s.* 11*d.* to halfpence.

(7) How many threepences in 796 guineas? and how many sixpences in 1592 guineas?

(8) Reduce £897 19*s.* 11¾*d.* to farthings, and £3591 19*s.* 7*d.* to pence.

(9) Bring 71016386 farthings to guineas, and 35508193 farthings to half-guineas.

(10) Reduce 37914 threepences to fourpences, and 568710 three halfpences to half-crowns.

(11) Bring £791 to guineas, and £395 10*s.* to half-guineas.

(12) How many farthings in 7965 guineas? and how many halfpence in 15930 guineas?

(13) The debt of England may amount to £799,817,516 18*s.* 10½*d*; to how many farthings is that equivalent?

(14) Bring £1599635033 17*s.* 9*d.* to halfpence.

Divide £799 16*s.* 6*d.* among 36 men, and state how many farthings each will receive in value.

(15) Divide 57591 half-crowns among 324 persons, and state how many farthings each will receive in value.

Reduce 7916 half-crowns to guineas, and 1979 crowns to half-guineas.

WEIGHTS AND MEASURES.

(16) In 79 lbs. 11 oz. 17 dwts. 13 grs., how many grains?

In 205 tons 13 cwt. 3 qrs. 1 lb., how many lbs?

(17) Bring 1724653 grs. to lbs. (Troy), and multiply 6 lbs. 1 oz. 6 dwts. 13 grs. by 49.

(18) Reduce 9483975 grs. to lbs., and multiply 65 lbs. 10 oz. 6 dwts. 17 grs. by 25.

(19) If 297 spoons weigh 339 grains each, what is the total weight in lbs., &c.? and what will 33 spoons, each 6 oz. 7 dwts. 3 grs. weigh?

(20) What is the weight in grs. of 9 ingots of gold, each 9 oz. 17 dwts.? Also, how many lbs. in 18 tons 19 cwt. 3 qrs. 20 lbs. (Avoir.)?

(21) How many grs. in 3 lbs. 5 oz. 7 drs. 2 sc. 17 grs.? and how many farthings in £20 19*s.* 11¼*d.*?

(22) How many lbs. in a hhd. of sugar which weighs 17 cwt. 3 qrs. 10 lbs.? and how many pence in £8 6*s.* 6*d.*?

(23) In 379 yds. 1 qr. 1 nl., how many nls.? and in £6 6*s.* 5¼*d.*, how many farthings?

(24) Reduce 79356 pints to gallons; and multiply 826 gals. 2 qts. 1 pt. by 12.

(25) How many half-pints in 379 butts of beer? and how many farthings in £85 5*s.* 6*d.*?

(26) Reduce 39 miles to inches; and £10296 to pence.

(27) In 71 acres 3 rds. 17 poles, how many poles? and in 2 miles 1 fur. 16 poles 4 yds. 1 ft., how many feet?

(28) Reduce 62523207 inches to miles; and multiply 12 miles 3 fur. 37 poles 2 ft. 3 in. by 79.

(29) In 37 cubic yards, how many cubic inches? and in £1798 4*s.* how many farthings?

(30) Bring 317376 cubic inches to yards; and multiply 7 ft. 1128 inches by 24.

(31) In 399 qrs. of corn, how many pts.? and in 157 sq. yds. 5 ft. 96 inches, how many inches?

(32) Reduce 537936 gallons to quarters; and multiply 175 qrs. 3 pks. 1 gal. by 48.

(33) In 791 years, how many days? and in £1203 16*s.* 0¾*d.* how many pence?

(34) Bring 9895716 minutes to days; and multiply 63 days 15 hours 7 minutes by 108.

(35) In 3171888 inches how many miles? and multiply 2 fur. 31 poles 1 yd. 1 ft. 1 in. by 144.

DIVISION OF COMPOUND QUANTITIES.

This is to find how many times one compound quantity is contained in another; the quotient will be an abstract number, or merely 'so many times,' and the remainder, if any, will be of the same denomination as the dividend was reduced to.

Rule.—Bring the divisor and dividend to the same name, and divide the greater result by the less.

Exercise 30.

(1) Divide £48 19*s.* 6*d.* by £2 19*s.* 5*d.*; and also £122 8*s.* 9*d.* by £7 8*s.* 6½*d.*

(2) Divide £793 19*s.* 10½*d.* by £29 15*s.* 2¼*d.*; also £2646 12*s.* 11*d.* by £99 3*s.* 11½*d.*

(3) Divide £832 13*s.* by £89 10*s.* 6*d.*; and 8723 guineas by £984 15*s.* 6*d.*

(4) How many times is 3 qrs. 17 lbs. contained in 1 ton?
Divide 9 tons by 8 cwt. 13 lbs.

(5) Divide 79 tons 15 cwt. 2 qrs. 11 lbs. by 3 tons 9 cwt. 3 qrs. 10 lbs.; and 877 tons 11 cwt. 2 qrs. 9 lbs. by 38 tons 8 cwt. 0 qrs. 26 lbs.

(6) Divide 7916 days 17 hrs. 15 min. 19 sec. by 37 days 23 hrs. 11 min. 18 sec.; and 27708 days 12 hrs. 23 min. $36\frac{1}{2}$ sec. by 132 days 21 hrs. 9 min. 33 sec.

(7) Divide 763 qrs. 7 bus. 3 pks. by 3 bus. 3 pks. 1 gal.; also 8403 qrs. 5 bus. 1 pk. by 42 bus. 2 pks. 1 gal.

Divide 871 miles 5 fur. 17 poles by 29 miles 3 fur. 10 poles; also, 9588 miles 3 fur. 27 poles by 323 miles 3 fur. 30 poles.

Note.—In these sums the remainder of one sum will be a multiple of the remainder of the other; or rather, the small remainder should be contained an exact number of times in the larger.

COMMON MEASURE.

If one number is contained an exact number of times in another, the smaller number is said to be a measure or factor of that number. Unity is not considered a factor or measure.

When a certain number is contained an exact number of times in two or more numbers, the smaller number is said to be a common measure of the others.

It is usually desirable to find the greatest common measure, or the greatest number that will divide any other given numbers.

To find the greatest common measure of two numbers—

Rule.—Divide the greater by the less, and the first divisor by the remainder; and so on, dividing each divisor by its corresponding remainder until there is no remainder; the last divisor is the greatest common measure.

Exercise 31.

Find the greatest common measure (G. C. M.) of 1071 and 1827; also of 2331 and 3717.

Find the greatest common measure of 300 and 960; also of 1020 and 2220.

What is the G. C. M. of 3213 and 6399; also 3559 and 12393?

Find the G. C. M. of

3213	and	5481;	also of	6993	and 11151
60	and	192;	also of	204	and 444
177	and	381;	also of	3576	and 7551
2691	and	4653;	also of	10557	and 19179
3367	and	4823;	also of	1027	and 1261

LEAST COMMON MULTIPLE.

One number is said to be a multiple of another when it can be divided by it without a remainder; thus, 24 is a multiptle of 8, because 24 can be divided by 8, leaving no remainder.

A common multiple is a number that can be divided by any of several others, and the least common multiple of any other number is the smallest number which can be divided by all separately without leaving a remainder; thus, 60 is the least common multiple of 2, 3, 4, and 5, because 60 is the smallest number that can be divided by 2, 3, 4, or 5, taken separately.

The *least* common multiple is generally required.

To find the least common multiple of two or more numbers—

Having placed them in a horizontal line, cancel or cross out any of the small numbers contained in the longer; for this reason, that the multiple of a large number will always be the multiple of its factor; then divide as many of the cancelled numbers as possible by a common factor or measure, and bringing down those that cannot be divided; continue this operation till no two numbers are left that have a common measure; then multiply the divisors and undivided numbers together, and the product will be the least common multiple.

Example.—What is the least common multiple of 2, 4, 5, 8, 12 and 16.

Here I find that 2, 4, and 8 are contained in other numbers of the series, I therefore cancel them thus—2, 4, 5, 8, 12, 16; leaving 5, 12, and 16; I find that 12 and 16 are divisible by 4—

$$4)5,\ 12,\ 16$$
$$5,\ 3,\ 4$$
$$4 \times 5 \times 3 \times 4 = 240 \text{ L. C. M.}$$

Exercise 32.

Find the least common multiple of—

(1) 8, 12, 18 and 3, 6, 8, 9, 24
(2) 3, 5, 10, 20 and 4, 15, 30
(3) 8, 12, 25, 50 and 2, 3, 4, 5, 6, 10, 20, 30, 200
(4) 17, 25, 51, 102 and 3, 5, 10, 17, 50
(5) 4, 9, 12, 18, 20, 24 and 3, 5, 6, 20, 24, 9
(6) 18, 25, 30, 42, 28 and 3, 20, 25, 40, 63
(7) 25, 40, 84, 100, 120 and 3, 4, 7, 8, 20, 25

VULGAR FRACTIONS.

A fraction is a numerical expression of a part of a whole thing, unit, or integer.

Vulgar, applied to fractions, means the *ordinary* way of expressing numerical parts as distinct from decimal fractions. A vulgar fraction is expressed by two numbers, one placed above the other, and a line drawn between them.

The lower figure is called the Denominator, or 'Namer,' because it shows how many parts the unit is divided into; and the upper figure is called the Numerator, or 'Numberer,' because it shows the number of those parts that is to be taken.

Thus, $\frac{5}{6}$ means that some unit has been divided or supposed to be divided into sixths, or six equal parts and 5 are meant to be taken. Fractional quantities are of various kinds, as Proper, Improper, Compound, Complex, &c.

A proper fraction has the numerator less than the denominator, and is always less than unity, as $\frac{3}{4}$, $\frac{9}{11}$.

An improper fraction has the numerator greater than the denominator, and is always greater than unity, as $\frac{4}{3}$, $\frac{11}{9}$.

A compound fraction is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{5}$ of $\frac{5}{9}$ of $\frac{7}{11}$.

A complex fraction has a fraction for its numerator or denominator, or for both, as $\frac{\frac{2}{3}}{7}$, $\frac{3}{\frac{2}{9}}$, $\frac{\frac{2}{5}}{\frac{3}{11}}$, $\frac{\frac{3}{5} \text{ of } 3\frac{1}{2}}{\frac{6}{9} \text{ of } 100}$.

A mixed number is a quantity that contains a whole number and a fraction, as $3\frac{2}{5}$, $4\frac{1}{9}$, &c.

To express a whole number by an equivalent fraction, put the given number as a numerator and 1 for a denominator.

CASE 1.—To reduce a simple fraction to its lowest terms; that is, to express it in the lowest figures—

Rule.—Divide the numerator and denominator by their common factors, which, with ordinary fractions, may easily be found; if the fraction is large, a quicker method is to find one common factor or measure, as has been already shown, and then to divide both parts of the fraction by this factor. Thus, $\frac{4)256}{528} = \frac{4)64}{132} = \frac{16}{33}$.

EXERCISE 33.

(1)	$\frac{91}{117}$	and	$\frac{105}{135}$	(6)	$\frac{365}{584}$	and	$\frac{475}{760}$
(2)	$\frac{585}{819}$	and	$\frac{1070}{1498}$	(7)	$\frac{4181}{4329}$	and	$\frac{1808}{1872}$
(3)	$\frac{493}{595}$	and	$\frac{609}{735}$	(8)	$\frac{3627}{8029}$	and	$\frac{34281}{75887}$
(4)	$\frac{361}{684}$	and	$\frac{133}{252}$	(9)	$\frac{32643}{72261}$	and	$\frac{239967}{531209}$
(5)	$\frac{72}{88}$	and	$\frac{144}{176}$	(10)	$\frac{7395}{8925}$	and	$\frac{4263}{5145}$

(11)	$\frac{927}{11303}$	and	$\frac{1314}{1602}$	(13)	$\frac{60071}{169807}$	and	$\frac{73787}{208579}$
(12)	$\frac{27813}{33909}$	and	$\frac{14454}{17622}$	(14)	$\frac{83439}{101727}$	and	$\frac{130086}{158598}$

CASE 2.—To reduce a mixed number to an improper fraction—

Rule.—Multiply the whole part of the quantity by the denominator of the fraction, and add thereto the numerator, placing the sum thus found over the denominator of the fractional part for the answer; thus, to reduce $3\frac{2}{7}$ to an improper fraction—

$$\frac{3 \times 7 + 2}{7} = \frac{23}{7} \text{ Ans.}$$

N.B.—In these questions the second of each set will have a different denominator for the answer.

EXERCISE 34.

Reduce to improper fractions—

(1)	$5\frac{3}{7}$	and	$7\frac{3}{5}$	(6)	$23\frac{3}{7}$	and	$18\frac{2}{9}$
(2)	$17\frac{3}{11}$	and	$14\frac{8}{13}$	(7)	$59\frac{5}{9}$	and	$48\frac{8}{11}$
(3)	$79\frac{16}{19}$	and	$65\frac{22}{23}$	(8)	$329\frac{3}{5}$	and	$56\frac{24}{29}$
(4)	$273\frac{5}{19}$	and	$179\frac{1}{99}$	(9)	$119\frac{27}{30}$	and	$59\frac{7}{76}$
(5)	$4\frac{13}{15}$	and	$14\frac{3}{5}$	(10)	$237\frac{112}{273}$	and	$407\frac{100}{139}$

CASE 3.—To reduce an improper fraction to a whole or mixed number—

Rule.—Divide the numerator by the denominator; the quotient will be a whole number, the remainder, if any, must be placed over the denominator of the given quantity and annexed to the quotient for the answer; thus, to reduce $\frac{17}{9}$ to a mixed number—

$$17 \div 9 = 1 \text{ and } 8 \text{ over } = 1\frac{8}{9} \text{ Ans.}$$

N.B.—In each of the following sets the whole numbers and numerators should turn out the same; the denominators will differ.

Exercise 35.

Reduce to mixed numbers—

(1)	$\frac{37}{5}$	and	$\frac{65}{9}$	(5)	$\frac{117}{12}$	and	$\frac{108}{11}$
(2)	$\frac{127}{11}$	and	$\frac{149}{13}$	(6)	$\frac{594}{13}$	and	$\frac{774}{17}$
(3)	$\frac{235}{12}$	and	$\frac{330}{17}$	(7)	$\frac{895}{23}$	and	$\frac{2263}{59}$
(4)	$\frac{3149}{275}$	and	$\frac{3721}{327}$	(8)	$\frac{8197}{57}$	and	$\frac{9627}{67}$

Case 4.—To reduce a compound fraction to a simple one—

Rule.—Simplify the fractional quantities, that is, reduce mixed numbers to improper fractions ; any whole numbers may be expressed fractionally by making 1 the denominator; then place them in a line with a multiplication sign between them. Multiply the numerators for a new numerator and the denominators for a new denominator.

If any of the numerators and denominators have common factors, divide them by such factors, remembering that for every numerator divided one denominator must be divided also by the same factor.

Example.—Reduce $\frac{3}{5}$ of $\frac{10}{21}$ of $\frac{7}{12}$ to a simple fraction by cancelling—

$$\frac{\cancel{3}^{1}}{\cancel{5}} \times \frac{\cancel{10}^{\cancel{2}}}{\cancel{21}_{\cancel{7}}} \times \frac{\cancel{7}^{1}}{\cancel{12}_{6}} = \frac{1}{6} \quad \textit{Ans.}$$

Exercise 36.

Express as simple fractions—

(1)	$\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{7}$	and	$\frac{3}{5} \times \frac{5}{7} \times \frac{8}{9}$
(2)	$\frac{2}{4}$ of $\frac{3}{7}$ of $5\frac{1}{8}$	and	$1\frac{3}{4} \times 2\frac{11}{16}$ of $\frac{2}{7}$
(3)	$\frac{1}{3}$ of $\frac{3}{4}$ of $5\frac{1}{2}$	and	$\frac{4}{33}$ of $1\frac{1}{7}$ of $2\frac{3}{4}$
(4)	$2\frac{1}{3}$ of 9 of $\frac{7}{11}$ of $\frac{3}{5}$	and	$5\frac{5}{6}$ of $3\frac{3}{5}$ of $\frac{28}{33}$ of $\frac{9}{10}$
(5)	$1\frac{1}{3}$ of 10 $\frac{1}{1}$ of $\frac{100}{12}$	and	$\frac{45}{56}$ of $14\frac{7}{25}$ of $\frac{20}{36}$ of 15

(6) $\frac{11}{14}$ of $3\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{17}$ and $5\frac{1}{2}$ of $\frac{11}{21}$ of $\frac{3}{11}$ of $\frac{42}{51}$
(7) $\frac{117}{125}$ of $\frac{213}{125}$ of $\frac{59}{147}$ and $3\frac{221}{420}$ of $\frac{428}{285}$ of $\frac{59}{532}$
(8) $1\frac{1}{3}$ of $\frac{3}{7}$ of $3\frac{1}{4}$ of $\frac{5}{4}$ of 7 of $1\frac{1}{5}$ and $\frac{1}{21}$ of $4\frac{1}{16}$ + 63
(9) $\frac{3}{11}$ of $15\frac{2}{3}$ of $4\frac{1}{2}$ and $4\frac{3}{11}$ of $3\frac{1}{2}$ of $1\frac{2}{7}$

CASE 5.—To reduce fractions to a common denominator—

Rule 1.—Having reduced each quantity to a simple form, that is to proper or improper fractions, multiply each numerator by all the denominators except its own, and multiply all the denominators for a new denominator; or

Rule 2.—Find the least common multiple of all the denominators; divide this by each denominator separately, and multiply the quotient by the corresponding numerators for new numerators, under each of which place the least common multiple. This is the most useful method of bringing fractions to a common denominator.

By *Rule* 1.

Bring $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{1}{7}$ to a common denominator—

$2 \times 5 \times 7 = 70$
$3 \times 3 \times 7 = 63$
$1 \times 3 \times 5 = 15$

$3 \times 5 \times 7 = 105$

$\frac{70}{105}$, $\frac{63}{105}$, $\frac{15}{105}$

By *Rule* 2.

Bring $\frac{2}{15}$, $\frac{5}{20}$, $\frac{13}{16}$, to a common denominator—

2)18 20 16
2)9 10 8
9 5 4

$2 \times 2 \times 9 \times 5 \times 4 = 720$
$720 \div 18 \times 3 = 120$
$720 \div 20 \times 5 = 180$
$720 \div 16 \times 13 = 585$

$\frac{120}{720}$, $\frac{180}{720}$, $\frac{585}{720}$ *Ans.*

In the following examples the numerators only will correspond when each set is reduced to the least common denominator:—

EXERCISE 37.

Reduce to the least common denominator—

(1) $\frac{3}{5}$, $\frac{3}{4}$, $\frac{4}{9}$, and $\frac{5}{7}$, $\frac{9}{16}$, $\frac{13}{85}$
(2) $1\frac{3}{8}$, $3\frac{1}{3}$, $4\frac{1}{6}$ and $\frac{3}{16}$, $\frac{10}{37}$, $\frac{17}{85}$

(3) $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{20}$, and $\frac{5}{6}$, $\frac{7}{9}$, $\frac{3}{5}$

(4) $\frac{3}{7}$, $\frac{5}{12}$, $\frac{9}{20}$, $\frac{5}{40}$, and $\frac{4}{7}$, $\frac{5}{9}$, $\frac{1}{6}$, $\frac{1}{8}$

(5) $2\frac{1}{3}$, $1\frac{1}{10}$, $\frac{5}{9}$, $\frac{3}{7}$, and $2\frac{1}{10}$, $\frac{99}{100}$, $\frac{9}{18}$, $\frac{27}{70}$

(6) $1\frac{1}{2}$ of $3\frac{1}{3}$, $\frac{15}{7}$, 9, $\frac{4}{5}$, and $\frac{3}{4}$, $\frac{15}{49}$, $\frac{9}{7}$, $\frac{4}{35}$

(7) $\frac{4}{5}$, $\frac{9}{10}$, $\frac{13}{20}$, $\frac{11}{30}$, $\frac{21}{10}$, and $\frac{2}{3}$, $\frac{3}{4}$, $\frac{13}{24}$, $\frac{11}{36}$, $\frac{7}{4}$

(8) $\frac{13}{14}$, $\frac{17}{39}$, $\frac{23}{54}$, and $1\frac{5}{21}$, $\frac{17}{24}$, $\frac{46}{81}$

ADDITION OF FRACTIONS.

Rule.—Simplify the quantities, if necessary, as before directed; bring them to the same denominator by the last case; then add the numerators together for a new numerator and place it over the common denominator; if the result be an improper fraction reduce it to a mixed number.

When any of the quantities to be added are mixed numbers, add the fractional parts together first, and the sum of the whole numbers can be added to this result for the final total.

EXAMPLE 1.—Add $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{7}$.

$$\begin{aligned} 2 \times 5 \times 7 &= 70 \\ 3 \times 3 \times 7 &= 63 \\ 4 \times 3 \times 5 &= 60 \\ \hline 3 \times 5 \times 7 &= 105 \end{aligned} \qquad \frac{70+63+60}{105} = \frac{193}{105} = 1\frac{88}{105} \textit{ Sum.}$$

EXAMPLE 2.—Add $3\frac{1}{4}$, $4\frac{1}{2}$, $5\frac{1}{3}$ together.

$$\frac{1}{4} + \frac{1}{2} + \frac{1}{3} = \frac{3+6+4}{12} = \frac{13}{12} = 1\frac{1}{12}$$

$$3 + 4 + 5 + 1\frac{1}{12} = 13\frac{1}{12} \textit{ Ans.}$$

N.B.—If the given quantities have the same denominator, no previous reduction is required, merely add the numbers as before directed and place over the denominator; reduce them if necessary.

EXERCISE 38.

Find the value of

(1) $\frac{3}{5}$, $\frac{7}{5}$, $2\frac{1}{5}$, 9, also of $\frac{7}{5}$, $\frac{11}{5} + 6\frac{1}{5} + 5$

(2) $\frac{2}{3} + 1\frac{1}{3} + \frac{5}{3} + 3$, also of $\frac{7}{12} + 1\frac{11}{12} + 4\frac{1}{12} + \frac{1}{12}$

(3) $\frac{5}{6} + \frac{7}{8}$ also of $1\frac{1}{8} + \frac{13}{24}$

(4) $3\frac{1}{5} + 5\frac{1}{3}$ also of $\frac{43}{15} + \frac{17}{3}$

(5) $\frac{3}{5} + \frac{2}{11} + \frac{1}{3}$ also of $\frac{4}{15} + \frac{3}{77} + \frac{41}{42}$

(6) $3\frac{1}{3} + 2\frac{1}{5} + 4\frac{1}{2}$ also of $2\frac{5}{6} + 6\frac{7}{5} + \frac{7}{10}$

(7) $\frac{7}{9} + \frac{3}{7} + \frac{2}{5}$ also of $\frac{4}{9}$, $\frac{13}{21}$, $\frac{19}{35}$

(8) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ also of $\frac{11}{24} + \frac{2}{5} + \frac{9}{20}$

(9) $\frac{1}{2} + \frac{1}{3} + \frac{4}{5} + \frac{8}{9}$ also of $\frac{13}{18} + 1\frac{31}{45} + \frac{1}{9}$

(10) $\frac{3}{7} + 2\frac{1}{4} + \frac{5}{11}$ also of $3 + \frac{1}{84} + \frac{4}{33}$

SUBTRACTION OF FRACTIONS.

Rule.—Prepare the numbers, if necessary, by bringing them to the least common denominator; then subtract the less numerator from the greater, and place the difference over the common denominator for the answer.

If the quantities contain whole numbers, subtract the fractional parts and annex the result to the difference of the whole numbers.

EXAMPLE 1.—From $\frac{7}{9}$ take $\frac{3}{9}$

$$\frac{7-3}{9} = \frac{4}{9} \quad \textit{Ans.}$$

EXAMPLE 2.—From $\frac{5}{7}$ take $\frac{3}{5}$

35 = L.C.M. of denominators.

$\frac{25}{35} - \frac{21}{35} = \frac{4}{35}$ *Ans.*

EXAMPLE 3.—From $3\frac{1}{7}$ take $2\frac{2}{3}$.

$\frac{2}{3}$ cannot be taken from $\frac{1}{7}$; I therefore borrow a unit or seven-sevenths, which being added make

$$\frac{8}{7}, \text{ then } \frac{8}{7} - \frac{2}{3} = \frac{24-14}{21} = \frac{10}{21}$$

now adding 1 borrowed to the lower whole number makes 3, and 3 from 3 remains 0; therefore the difference is $\frac{10}{21}$.

EXERCISE 39

Find the values of the following expressions—

(1) $\frac{11}{19} - \frac{6}{19}$ and $\frac{9}{14} - \frac{3}{14}$.

(2) $\frac{17}{18} - \frac{3}{18}$ and $\frac{20}{9} - 1\frac{4}{9}$.

(3) $\frac{1}{4} - \frac{1}{5}$ and $\frac{7}{20} - \frac{3}{10}$

(4) $\frac{8}{9} - \frac{3}{7}$ and $3\frac{1}{8} - 2\frac{55}{88}$

(5) $3\frac{1}{2} - 1\frac{1}{3}$ and $5\frac{1}{7} - 2\frac{11}{42}$

(6) $17\frac{9}{10} - 10\frac{3}{5}$ and $19\frac{3}{14} - 12\frac{11}{28}$

(7) $25\frac{1}{2} - 17\frac{1}{3}$ and $19\frac{5}{6} - 11\frac{2}{3}$

(8) $\frac{11}{17} - \frac{7}{13}$ and $9 - 8\frac{197}{221}$

(9) $4\frac{1}{3} - 3\frac{1}{7}$ and $3\frac{7}{8} - 2\frac{37}{65}$

(10) $13\frac{9}{16} - 7\frac{1}{11}$ and $25\frac{1}{8} - 19\frac{5}{176}$.

MULTIPLICATION OF FRACTIONS.

Rule.—Simplify the quantities and multiply all the numerators together for a new numerator, and the denominators together for a new denominator.

Before multiplying, divide the numerators and denominators by any common factors, as in Case 4.

EXAMPLE 1.—Find the value of $\frac{2}{3} \times \frac{3}{4}$

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2} \quad \textit{Product.}$$

EXAMPLE 2.—Find the value of $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{2}{3}$

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{7}{\underset{4}{\cancel{8}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \frac{7}{60} \quad \textit{Ans.}$$

EXERCISE 40.

Find the value of—

(1) $\frac{3}{5} \times \frac{2}{3}$ also $\frac{2}{3} \times \frac{3}{15}$.
(2) $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ and $\frac{1}{2} \times 1\frac{1}{3} \times 10\frac{2}{7} \times \frac{1}{15}$.
(3) $2\frac{1}{3} \times 10\frac{1}{2} \times 3\frac{3}{5}$ and $3\frac{1}{4} \times 4\frac{2}{3} \times 5\frac{2}{5}$.
(4) $\frac{3}{13} \times 3\frac{3}{4} \times \frac{9}{14} \times 23\frac{1}{3}$ and $\frac{3}{13} \times 2\frac{1}{3} \times 1\frac{1}{2} \times 10$.
(5) $\frac{1}{7} \times \frac{3}{5} \times \frac{2}{3} \times \frac{4}{7}$ and $\frac{9}{77} \times \frac{11}{15} \times \frac{6}{11} \times \frac{44}{85}$.
(6) $\frac{3}{7}$ of $2\frac{1}{3} \times \frac{4}{5}$ of 9 and $\frac{16}{35}$ of $15\frac{3}{4} \times \frac{6}{7}$ of $1\frac{1}{6}$.
(7) $\frac{7}{9} \times 11\frac{1}{12} \times 10\frac{2}{3}$ and $\frac{49}{81} \times \frac{133}{12} \times 2\frac{4}{7} \times 5\frac{1}{3}$.
(8) $1\frac{1}{2} \times 2\frac{2}{3} \times 3\frac{3}{4} \times 4\frac{4}{5}$ and $\frac{9}{10} \times 13\frac{1}{3} \times 1\frac{1}{44} \times 5\frac{39}{45}$.
(9) $\frac{11}{17} \times \frac{18}{19} \times \frac{20}{21} \times \frac{23}{23}$ and $\frac{22}{51} \times \frac{27}{19} \times \frac{40}{33} \times \frac{33}{23}$.
(10) $17\frac{1}{2} \times 18\frac{1}{3} \times 14\frac{1}{4}$ and $\frac{175}{56} \times 25\frac{2}{3} \times 57$.

N.B.—To multiply a fraction by a whole number, either multiply the numerator, or divide the denominator.

EXERCISE 41.

(1)	Multiply	$\frac{35}{48}$	by	8	and	$\frac{35}{54}$	by	9
(2)	,,	$\frac{33}{32}$	,,	4	,,	$\frac{46}{126}$	,,	8
(3)	,,	$\frac{110}{225}$	,,	15	,,	$\frac{22}{27}$	,,	9
(4)	,,	$\frac{259}{192}$	,,	24	,,	$\frac{37}{8}$	,,	7
(5)	,,	$\frac{1809}{2596}$	,,	11	,,	$\frac{77}{236}$	,,	17
(6)	,,	$\frac{2010}{1886}$	,,	14	,,	$\frac{201}{112}$	,,	10

DIVISION OF FRACTIONS.

Rule.—Simplify the quantities if necessary, invert the divisor and multiply them.

EXAMPLE 1.—Divide $\frac{3}{4}$ by $\frac{5}{6}$

$$\frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10} \quad \textit{Ans.}$$

EXAMPLE 2.—Divide $\frac{1}{2}$ of $\frac{1}{3}$ by $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{1} \times \frac{4}{1} = \frac{4}{2} = 2$ *Ans.*

EXERCISE 42.

Find the value of—

(1) $\frac{3}{4} \div \frac{2}{5}$ and $5\frac{1}{4} \div 1\frac{5}{9}$
(2) $\frac{2}{11} \div 3\frac{1}{4}$ „ $\frac{56}{429} \div 2\frac{1}{3}$
(3) $3\frac{1}{4} \div 2\frac{1}{7}$ „ $\frac{13}{20} \div \frac{3}{7}$
(4) $22\frac{1}{8} \div 17\frac{1}{3}$ „ $\frac{1485}{1248} \div \frac{11}{12}$
(5) $\frac{3}{11} \div \frac{4}{5}$ „ $\frac{5}{22} \div \frac{2}{3}$
(6) $2\frac{1}{3}$ of $\frac{3}{7} \div \frac{3}{4}$ of 5 and $1 \div 3\frac{3}{4}$
(7) $\frac{5}{6}$ of $\frac{3}{7}$ of $\frac{2}{11} \div \frac{4}{13}$ and $\frac{10}{231} \div \frac{8}{91}$ of $3\frac{1}{2}$
(8) $2\frac{1}{3}$ of $3\frac{1}{4} \div 4\frac{1}{2}$ of $5\frac{1}{8}$ also $\frac{13}{12}$ by $\frac{23}{30}$

To divide a fraction by a whole number.

Multiply the denominator or divide the numerator by the given number.

EXAMPLE.—Divide $\frac{32}{40}$ by 8

$$\frac{32 \div 8}{40} = \frac{4}{40} = \frac{1}{10} \quad \textit{Ans.}$$

or $$\frac{32}{40 \times 8} = \frac{32}{320} = \frac{1}{10} \quad \textit{Ans.}$$

EXERCISE 43.

(1) Divide $\frac{20}{36}$ by 5 and $\frac{4}{3}$ by 12
(2) „ $\frac{17}{18}$ „ 9 „ $\frac{68}{162}$ „ 4
(3) „ $\frac{120}{121}$ „ 15 „ $\frac{8}{11}$ „ 11
(4) „ $\frac{19}{4}$ „ 36 „ $\frac{145}{144}$ „ 5
(5) „ $\frac{1002}{1251}$ „ 25 „ $\frac{40}{417}$ „ 3

COMPLEX FRACTIONS.

In every fraction the numerator is supposed to have the denominator for its divisor; it is not difficult, therefore, to

reduce any complex fraction to a simple one as it resolves itself merely into a division operation thus $\frac{\frac{2}{3}}{\frac{3}{4}}$ means really $\frac{2}{3}$ divided by $\frac{3}{4}$ which is done by the last rule.

Rule.—Simplify the numerator and denominator, that is, resolve each into simple fractions, invert those that compose the denominator, and multiply them into the parts of the numerator.

EXAMPLE.—Simplify $\frac{\frac{3}{5}}{\frac{2}{3}}$ and $\frac{\frac{2}{5}}{\frac{4}{5}}$

Here $\frac{3}{5}$ is to be divided by $\frac{2}{3}$ and $\frac{2}{5}$ by $\frac{4}{5}$; in each case $\frac{9}{10}$ is the result.

EXERCISE 44.

(1) Simplify $\frac{1\frac{1}{2}}{\frac{2}{3}\text{ of }\frac{4}{7}}$ and $\frac{\frac{3}{7}}{\frac{8}{21}\text{ of }\frac{14}{49}}$

(2) „ $\frac{2\frac{1}{3}\text{ of }3\frac{1}{4}}{13\frac{2}{5}}$ and $\frac{\frac{1}{6}\text{ of }15\frac{1}{6}}{3\frac{29}{35}+\frac{2}{7}}$

(3) „ $\frac{27}{3\frac{1}{5}}$ and $\frac{15}{1\frac{7}{9}}$

(4) „ $\frac{\frac{7}{13}}{2\frac{1}{6}}$ and $\frac{1\frac{19}{39}}{5\frac{1}{18}}$

(5) „ $\frac{2\frac{1}{6}}{2\frac{5}{9}}$ and $\frac{3\frac{1}{4}}{4\frac{1}{3}}$

(6) „ $\frac{3}{2\frac{1}{4}+1\frac{1}{2}}$ and $\frac{\frac{2}{7}\text{ of }10\frac{1}{2}}{\frac{2}{4}\text{ of }15}$

(7) „ $\frac{\frac{1}{3}\text{ of }\frac{1}{3}\text{ of }4}{\frac{2}{5}\text{ of }3\frac{\frac{1}{2}}{\frac{2}{7}}}$ and $\frac{\frac{4}{35}\div\frac{2}{5}}{(\frac{1}{21}\text{ of }\frac{61}{21})\div\frac{1}{7}}$

(8) „ $\frac{2\frac{1}{3}\text{ of }3\frac{2}{7}}{4\frac{1}{5}\text{ of }5\frac{2}{3}}$ and $\frac{7\frac{2}{3}}{76\frac{174}{195}\times\frac{13}{42}}$

REDUCTION OF FRACTIONS.

CASE 1.—*To reduce a fraction of one denomination to an equivalent fraction of a less or greater denomination.*

Rule.—If the fraction is to be reduced to a greater name,

multiply the denominator, but if to a less name, the numerator, by as many of the less denomination as make one of the denomination sought—the product will be the fraction required.

EXAMPLE 1.—Reduce $\frac{2}{3}$ of a shilling to the fraction of a pound.

$$\frac{2}{3 \times 12 \times 20} = \frac{2}{720} = \frac{1}{360} \text{ £}$$

EXAMPLE 2.—Reduce $\frac{2}{5}$ of a guinea to the fraction of a penny.

$$\frac{2 \times 21 \times 12}{5} = \frac{504}{5} \text{ of a penny.}$$

EXERCISE 45.

(1) Reduce $\frac{5}{6}$ of a penny to the fraction of a £, and $\frac{1}{8}$ of a gallon beer to the fraction of a barrel.

(2) Reduce $\frac{3}{8}$ of a shilling to the fraction of a guinea, and $\frac{3}{7}$ of hour to the fraction of a day.

(3) Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of a lb. Avoir. to the fraction of a ton, and $\frac{3}{35}$ a pint to the fraction of a quarter (Dry Measure.)

(4) Reduce $\frac{3}{2100}$ of an oz. Troy, to the fraction of a grain, and $\frac{4}{33}$ a foot to the fraction of an inch.

(5) Reduce $\frac{1}{168}$ of a week to the fraction of a day, and $\frac{12}{3}$ of a lb. the fraction of a cwt.

CASE 2.—*To reduce one quantity to the fraction of another.*

Rule.—Reduce both quantities to the same denomination and place the former over the latter to form a fraction.

Thus, to reduce 2 qrs. 7 lbs. to the fraction of 3 qrs. 10 lbs.

2 qrs. 7 lbs.	3 qrs. 10 lbs.
28	28
63	94

$\frac{63}{94}$ *Ans.*

EXERCISE 46.

(1) Reduce 2*s.* 6*d.* to the fraction of a £, and 14 lbs. to the fraction of a cwt.

(2) Reduce 3*s.* 6*d.* to the fraction of a guinea and a half, and 3 qrs. 11 lbs. to the fraction of 7 cwt. 2 qrs. 15 lbs.

(3) Reduce 3 lb. 11 oz. to the fraction of a cwt., and 12 lbs. 14 oz. 8 drms. to the fraction of $3\frac{1}{2}$ cwt.

(4) Reduce 5 bus. 3 pks. to the fraction of a quarter, and 11 oz. 8 drms. to the fraction of a lb. (Avoir.)

(5) Reduce 3 days 10 hrs. 15 min. to the fraction of 7 days 11 hrs. 19 min.; also reduce 7 days 23 hrs. 55 min. to the fraction of 17 days 10 hrs. 24 min. 20 sec.

(6) Reduce 5 poles 27 yds. to the fraction of an acre, and 6 cwt. 1 qr. 13 lbs. to the fraction of 8 tons 12 cwt. 3 qrs. 12 lbs.

CASE 3.—*To reduce a fractional part of a given quantity to the fraction of another.*

Rule.—By the last case reduce the first quantity to a fraction of the second, and multiply this fraction by the fractional part prefixed, the result will be the answer.

EXAMPLE.—Reduce $\frac{3}{7}$ of a half-crown to the fraction of a guinea.

s.	*d.*	
2	6	1 guinea
2		42
5		42 sixpences

$\frac{3}{7} \times \frac{5}{42} = \frac{15}{294} = \frac{5}{98}$ *Ans.*

EXERCISE 47.

(1) Reduce $\frac{3}{11}$ of 3*s.* 4*d.* to the fraction of a £, and $\frac{1}{3}$ of 2 qrs. 21 lbs. to the fraction of 7 cwt. 2 qrs. 7 lbs.

(2) Reduce $\frac{14}{17}$ of 3 oz. 15 dwts. 10 grs. to the fraction of 3 lbs. 10 oz., and $\frac{28}{51}$ of 2 oz. 10 dwts. $6\frac{2}{3}$ grs. to the fraction of 2 lbs. 6 oz. 13 dwts. 8 grs.

(3) Reduce $\frac{3}{5}$ of 3 hrs. 15 minutes to the fraction of $\frac{5}{11}$ of 17 days 11 hrs 18 minutes, and $1\frac{2}{5}$ of 7 hrs. 35 minutes to the fraction of $1\frac{2}{33}$ of 5 wks. 5 days 18 hrs. 22 minutes.

(4) Reduce $\frac{4}{5}$ of £2 16*s.* 3*d.* to the fraction of £7 19*s.* 10*d.*, and 4 cwt. 3 qrs. 8 lbs. to the fraction of 17 cwt. 14 lbs.

MULTIPLICATION AND DIVISION OF COMPOUND QUANTITIES BY FRACTIONAL NUMBERS.

Rule for Multiplication.—Multiply by the numerator and divide by the denominator.

Rule for Division.—Multiply by the denominator and divide by the numerator, any remainders to be reduced and divided again.

EXAMPLE.—Find the value of $\frac{5}{7}$ of 3*s.* 6*d.*, and divide 3 qrs. 5 lbs. by $\frac{5}{6}$.

s.	*d.*		qrs.	lbs.			
3	6		3	5			
	5			6			
7)17	6		5)4	3	2		
2	6	*Ans.*		3	22	12 oz.	$12\frac{4}{5}$ drs.

EXERCISE 48.

Find the value of—

(1) $\frac{3}{4}$ of 1 ton, and $\frac{9}{32}$ of 2 tons 13 cwt. 1 qr. $9\frac{1}{3}$ lbs.

(2) $\frac{5}{7}$ of £4 10*s.* 0*d.* and $\frac{20}{21}$ of £3 7*s.* 6*d.*

(3) £79 + $\frac{7}{5}$, and £908 10*s.* 0*d.* ÷ $8\frac{17}{18}$.

(4) $\frac{5}{11}$ of a mile, and 2 fur. $22\frac{2}{3}$ poles × $1\frac{37}{88}$.

(5) 37 a. 3 r. $10\frac{1}{4}$ poles ÷ $2\frac{3}{5}$, and 50 a. 1 r. 27 poles ÷ $3\frac{7}{15}$.

(6) 793 guineas × $\frac{15}{16}$, and £208 3*s.* 4*d.* × $3\frac{3}{4}$.

(7) 3 cwt. 3 qrs. 15 lbs. × $3\frac{3}{5}$, and 2 tons 8 cwt. 3 qrs. 21 lbs. + $\frac{9}{7}$.

(8) £37 16*s.* 4*d.* × $5\frac{7}{8}$, and £777 12*s.* $1\frac{1}{4}d.$ + $3\frac{1}{2}$.

(9) $\frac{4}{7}$ of a cwt., and 22 cwt. 3 qrs. 12 lbs. + 40.

(10) 22 days 17 hrs. 59 m. 37 s. × $3\frac{1}{5}$, 113 days 17 hrs. 58 m. 5 s. × $\frac{7}{10}$.

MISCELLANEOUS QUESTIONS IN VULGAR FRACTIONS.

EXERCISE 49.

(1) Reduce $\frac{1053}{3331}$ and $\frac{2223}{4681}$ to their lowest terms.

(2) From $4\frac{5}{6}$ take $\frac{3}{7}$ of $2\frac{1}{3}$; and divide $4\frac{4}{63}$ by $1\frac{1}{7}$.

(3) What is the value of $\frac{3}{8}$ of 5 tons? and multiply 3 cwt. 1 qr. 11 lbs. by $11\frac{1}{3}$.

(4) What is the sum of $\frac{2}{3}$, $3\frac{1}{4}$, and $2\frac{3}{5}$? and what is the difference between $8\frac{11}{12}$ and $2\frac{3}{5}$?

5) Divide $\frac{3}{4}$ of $\frac{5}{7}$ by $\frac{1}{8} - \frac{5}{8}$; and multiply $2\frac{1}{7}$ by $1\frac{3}{7}$.

6) Sold $\frac{3}{4}$ of $\frac{5}{6}$ of a piece of cloth which measured 28 yards, how ch is left? I also sold another piece for 35*s.* at the rate of 3*s.* 4*d.* yard, how much was there?

7) What part of 3 cwt. 1 qr. 10 lbs. is 3 qrs. 7 lbs.? and what part 5*s.* 7*d.* is 3*s.* $9\frac{1}{2}$*d.*?

8) If I spend $\frac{3}{4}$ of half-a-crown and then $\frac{1}{5}$ of the remainder, how ch have I left? and what is the value of $\frac{3}{4}$ of $\frac{1}{7}$ of 2*s.* 4*d.*?

9) Take from 3 its fourth, eighth, and sixteenth parts, and multiply remainder by the difference between the product of $\frac{3}{8}$ and $2\frac{1}{3}$ and sum of $\frac{3}{8}$ and $\frac{3}{4}$; also, find the difference between the sum of $\frac{5}{6}$ and d the product of $1\frac{1}{3}$ and $\frac{7}{8}$, multiply it by the sixteenth part of 33, divide that product by $23\frac{6}{27}$.

10) Three-sevenths of a vessel are worth £1,240 13*s.* $11\frac{1}{4}$*d.*, what is value of the vessel? and multiply £701 16*s.* 2*d.* by $4\frac{1}{8}$

11) Won $\frac{1}{8}$ of $\frac{5}{6}$ of a pound and lost $\frac{3}{5}$ of $\frac{1}{13}$ of a guinea, how much I gain or lose? also find the 233rd part of £10 13*s.* 7*d.*

12) What is the sum of $13\frac{3}{8}$ and $9\frac{3}{7}$? also of $10\frac{7}{55}$ and $12\frac{54}{77}$?

13) What is the quotient of $20\frac{3}{10}$ divided by $4\frac{5}{6}$? also of $1\frac{10}{11}$ divided $\frac{5}{11}$?

14) Add together 27 cwt. 3 qrs. $15\frac{1}{4}$ lbs., 19 cwt. 2 qrs. $20\frac{3}{8}$ lbs., cwt. 1 qr. $15\frac{5}{6}$ lbs., and 9 cwt. 1 qr. $13\frac{3}{4}$ lbs.; also, add together cwt. 2 qrs. $14\frac{3}{5}$ lbs., 27 cwt. 2 qrs. $7\frac{5}{8}$ lbs., 19 cwt. 3 qrs. $1\frac{7}{12}$ lbs., and t. 1 qr. $13\frac{2}{5}$ lbs.

15) Add $\frac{3}{5}$ of a cwt. to $\frac{3}{5}$ of a ton; and add $6\frac{5}{21}$ cwt. to $7\frac{3}{5}$ cwt.

16) What fraction is 2*s.* $1\frac{3}{4}$*d.* of 5*s.* $7\frac{1}{2}$*d.*? and what fraction is s. 19 lbs. of 2 cwt. 1 qr. 18 lbs.?

17) If 1 yd. of silk be worth £$1\frac{11}{48}$, what is the worth of $37\frac{11}{12}$ yds.? the value of $113\frac{3}{4}$ yds. at $8\frac{7}{36}$*s.*?

18) Three-elevenths of a post are in the bed of a river, $\frac{5}{8}$ in the er, and there are 9 feet out the water, what is the length of the ? also, simplify $\frac{9}{17}$ of $\frac{3\frac{1}{4} - 1}{\frac{1}{44}}$ feet.

19) A sovereign weighs 5 dwts. $3\frac{171}{623}$ grains, what will be the weight bs. of $797\frac{1}{2}$ sovereigns? also, divide 61248000 grains by 623.

20) A man buys four pieces of cloth measuring together 36 yds.; a one piece he cuts $7\frac{3}{8}$ yds., from another $11\frac{7}{8}$ yds., from another rds., and from the other 3 yds. 1 qr. 1 nl., how much cloth has he altogether? also, divide 35223 nails by 597.

DECIMAL FRACTIONS.

The word decimal means tenth; decimal fractions are therefore the tenths, hundredths, etc., of an integer. The denominator of a decimal fraction being always a tenth or a power of it, it is unnecessary to express it as in vulgar fractions; but the value of the fraction is fixed by a dot or point placed before the first figure of the quantity, thus $\frac{3}{10}$, which is a decimal quantity, is better and shorter expressed by ·3; $\frac{3}{100}$ by ·03; $\frac{3}{1000}$ by ·003, etc. It will thus be seen that a great saving of figures is effected, and also that any decimal quantity may be expressed fractionally by putting 1 for the denominator with as many ciphers annexed as there are figures to the right of the decimal point; the dot and unnecessary ciphers may then be effaced; thus ·0014 is expressed by $\frac{14}{10000}$.

The power of a number is produced by multiplying it by itself any number of times: thus the first power of 8 is 64; of $\frac{1}{10}$, $\frac{1}{100}$.

DECIMAL NUMERATION.

Let it be noticed that the figure immediately to the right of a decimal point is tenths, the next hundredths, and so on decreasing by tenths; there are no units in decimals, thus ·357 is 3 tenths, 5 hundredths, 7 thousandths, which added together equal $\frac{357}{1000}$.

A cipher to the right of a decimal has no value or effect, thus ·300 is the same as ·3, but putting ciphers to the left after the point decreases it by tenths, thus ·5, ·05, ·005 equal $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$ respectively.

Express as decimals $\frac{3}{10}$, $\frac{15}{100}$, $\frac{123}{1000}$, $\frac{89}{100}$, $7\frac{15}{100}$, $\frac{23}{100000}$, $\frac{7}{100}$, $14\frac{11}{1000}$, $\frac{2}{50}$ of $\frac{9}{50}$, $\frac{7}{10}$ of $\frac{7}{100}$ of $\frac{7}{1000}$, $\frac{115}{10000}$.

Express as vulgar fractions ·4, ·09, ·001, ·0101, 3·45, ·10101, ·313154, 51 0013, ·64, 1·11010.

DECIMAL ADDITION AND SUBTRACTION.

Place the quantities to be added in such order under each other that the decimal points may be in the same vertical line and it is advisable to fill up the blank spaces with ciphers so as to make the right hand column of figures complete, then add as in common addition. Thus—

EXAMPLE.—Add together 3·1, 14·27, 9·3064, 4·51013.

3·10000
14·27000
9·30640
4·51013
31·18653 *Ans.*

In subtracting, observe the same rule as to the placing of the quantities, then proceed as in ordinary subtraction, Thus

EXAMPLE.—Take 3·101 from 4·0016.

4·0016
3·101
·9006 *Ans.*

EXERCISE 50.

(1) What is the sum of 37·15 + ·0014 + 1·76 + 31·534 + ·0004 + ·157 ? and the difference of 149·0136 + 17·51 and 23·5492 + 72·3716 ?

(2) Required the sum of 1·173 + 3·000456 + 297 + 3·7 + ·5 + ·1426 ; and the difference of 319·713056 and 14·197.

(3) Add 33·101 + 100·0015 + 2·76 + 9 + 1·843 + 375·1896 ; and subtract 389·78 from 911·6751.

(4) Add 2·710 + 10·01 + ·0054 + ·37962 + 28·197 + 5 ; and take 1·019 from 47·32102.

(5) Find the value of 7 + 9·103 + 275·1634 + 100·1001 + 57 + 3·920196 ; and the difference of 591·006222 and 138·719526.

(6) Find the sum of 2·713 + 12·1643 + ·0043 + 9·997 + 1010·1034 + 53 ; and the difference of 101·397 and 1189·379.

(7) Add 37·136 + ·0024 + 10·134 + 1·562 + 37·1015 + 55·6478 ; and take 79·517 from 221·1007.

(8) Add 173·1045 + 289·37 + 3816·0014 + 35·61 + ·10134 + 10·3796 ; also find the difference of 6220·94802 and 1896·38118.

(9) Add 117·1364 + ·0046 + 10·273 + 13·8196 + 27·1345 + 100·1296; and find the difference of 441·6513 and 173·1536.

DECIMAL MULTIPLICATION.

Multiply as in simple multiplication, and fix the value of the product by making as many decimal places in it as there are in the multiplicand and multiplier together. If there are not so many figures as decimal places required, supply the difference by prefixing ciphers. Thus: —

EXAMPLE.—Multiply 3·27 by ·004

3·27
·004
·01308

There are two decimal places in the multiplicand, and three in the multiplier; therefore mark off five decimals in the product; now as there are only four figures, a cipher must be added to make up the five.

EXERCISE 51.

(1) Multiply 37·164 by 2·891; also 260·148 by ·413.
(2) Multiply 18·195 by ·2103; also 54·585 by ·0701.
(3) Multiply 189·165 by 73·647; and 170248·5 by ·08183.
(4) Multiply 26·785 by ·0004; also 243·5 by ·000044.
(5) Multiply ·3796 by ·4185; also ·7592 by ·20925.
(6) Multiply 191·01017 by 101·1008; also 95·505085 by 202·2016.
(7) Find the product of ·079, ·086, and ·057; also of ·0158, 1·29, and ·019.
(8) Find the product of 7·35, ·085, and 73·975; also of 36·75, ·017, 6725, and ·011.

DECIMAL DIVISION.

The working is the same as in simple short or long division. When there are not enough figures in the dividend to bring down, supply ciphers, and in marking off the quotient, that is, putting the decimal point, count the number of decimal places in the dividend together with any ciphers you may have brought down, which of course are decimals:

then count the number of decimal places in the divisor and ascertain which has the most—the dividend with the ciphers brought down, or the divisor—and the difference will give the number of decimal figures you must mark off for the quotient.

Divide 3·72 by 1·36—

```
1·36)3·72(2·735
     272
     ----
     1000
      952
     ----
      480
      408
     ----
      720
      680
     ----
       40
     ====
```

In this example three ciphers are brought down, which, with the two decimal figures in the dividend, make 5. There are 2 in the quotient —2 from 5 leave 3—mark off then 3 places in the quotient.

N.B.—In these examples the greater remainder should be a multiple of the less.

Exercise 52.

(1) Divide 31·97 by 4·7 ; also 1·5985 by ·235.
(2) Divide 7·064 by ·86 ; also 63·576 by 7·74.
(3) Divide ·7186 by 3·958 ; also ·0014372 by ·007916.
(4) Divide 373·3 by 579·9 ; also 2·6131 by 4·0593.
(5) Divide 1018·35 by ·4962 ; also 339·45 by ·1654.
(6) Divide 1·01 by 101013 ; also ·00202 by 202·026.
(7) Divide 37·1968 by 589·9194 ; also 409·1648 by 6489·1134.
(8) Divide ·4088645 by ·00005258 ; also 37·1695 by ·00478.
(9) Divide 1·1963 by 27·163542 ; also ·1709 by 3·880506.

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal.

Rule.—Divide the numerator by the denominator, annexing ciphers to the former ; and the quotient, properly pointed according to the rule for division, will be the answer.

Thus; to reduce $\frac{2}{5}$ to a decimal, put down 2 with a cipher annexed.

$$5\overline{)2\cdot0}$$
$$\cdot4 \quad \textit{Ans.}$$

Before dividing the numerator by the denominator, reduce the fraction to its lowest terms.

In the following examples, reduce the first two of each set to decimals, add them together, and then prove by finding the decimal of the third.

Exercise 53.

(1) $\frac{3}{4}$, $\frac{1}{2}$, and $1\frac{1}{4}$; $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
(2) $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$; $\frac{3}{5}$, $\frac{4}{5}$, and $\frac{47}{40}$.
(3) $\frac{5}{8}$, $\frac{5}{12}$, and $1\frac{1}{24}$; $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{3}{8}$.
(4) $\frac{15}{16}$, $\frac{23}{24}$, and $1\frac{43}{48}$; $\frac{15}{1000}$, $\frac{7}{100}$, and $\frac{85}{1000}$.
(5) $\frac{2}{3}$ of $\frac{3}{4}$, $1\frac{1}{2}$ and $\frac{34}{17}$; $\frac{5}{6}$, $\frac{15}{4}$, and $4\frac{7}{12}$.
(6) $1\frac{4}{16}$, $2\frac{1}{4}$, and $3\frac{31}{80}$; $\frac{113}{452}$, $\frac{51}{250}$ and $\frac{115}{250}$.
(7) $\frac{351}{468}$, $\frac{595}{952}$, and $1\frac{87}{252}$; $\frac{17}{40}$, $\frac{13}{50}$, and $\frac{137}{200}$.

N.B.—In reducing a fraction to a decimal, if any remainder occur, twice the quotient figures will be the same as from the first of the two similar remainders, and the decimal will be a repeating or circulating decimal. (See next Case.)

OF CIRCULATING DECIMALS.

As it is sometimes impossible to divide the numerator by the denominator without leaving a remainder, or so as to get a perfect quotient, it will be seen that decimals in some cases are imperfect numbers, thus $\frac{2}{3}$ cannot be expressed by a complete decimal, for on division it will be found always to give the same remainder, and the quotient is ·6666 ad infinitum. To show that the figure 6 repeats, a dot is placed over it as $\dot{6}$.

A fraction will always produce a repeating decimal if, when reduced to its lowest terms, the denominator contains any other factors than 2 or 5; thus $\frac{7}{12}$ gives $\cdot58\dot{3}$, and $\frac{5}{9}$ gives $\cdot\dot{5}$ because 12 and 9 are multiples of 3, whereas $\frac{9}{16}$ gives

·5625, a complete decimal, because 16 contains no factor but 2 or a multiple of it.

A decimal in which some of the figures are repeated is called a repeating, recurring, or circulating decimal, or a circulator, and the part that repeats is called the period or repetend.

Circulators are of two kinds, pure and mixed.

A circulator is pure when the repeating figure or figures begin immediately after the decimal point, as ·1̇1̇1̇ or ·0̇2̇0̇2̇0̇2̇.

A mixed circulator is when the repeating figures do not immediately follow the point, as ·034̇, in which only the 4 is repeated.

To reduce a pure circulator to a fraction.

Rule.—Place the figures contained in the decimal as the numerator with as many nines for the denominator as there are figures in the decimal quantity, thus ·3̇ expressed fractionally becomes $\frac{3}{9}$ or $\frac{1}{3}$; ·0̇3̇ is $\frac{3}{99}$; ·0̇1̇2̇3̇ is $\frac{123}{999}$.

To reduce a mixed circulator to a fraction.

Subtract the figures after the point which do not circulate from all the figures composing the decimal quantity, and place the difference as a numerator, and for the denominator place as many nines as there are figures that circulate with as many ciphers as there are figures that do not circulate.

If the decimal be preceded by a whole number, consider only the decimals in applying the above rule. The whole number with the fraction annexed will form a mixed number. Thus:—

Reduce ·135̇ to a fraction.

·135 − 1 = 134 numr.; denr. = 990

$\frac{134}{990} = \frac{67}{495}$ *Ans.*

Reduce 3·179̇ to a fraction.

179 − 17 = 162 numr.; denr. = 900

Prefix the 3: $3\frac{162}{900} = 3\frac{81}{450} = 3\frac{9}{50}$ *Ans.*

In the following examples reduce the first two quantities to fractions, add them together and the result will be the vulgar fraction in the example.

Exercise 54.

(1) $\cdot\dot{6} + \cdot\dot{9} = 1\frac{2}{3}$ (2) $\cdot\dot{2} + \cdot\dot{1}\dot{8} = \frac{42}{99}$
(3) $\cdot 8 + \cdot 1\dot{2} = 1\frac{1}{90}$ (4) $\cdot 15 + 2\cdot\dot{6} = 2\frac{37}{45}$
(5) $2\cdot 3\dot{6} + 5\cdot 1\dot{4} = 7\frac{50}{99}$ (6) $\cdot 213 + \cdot 32\dot{7} = \frac{107}{198}$

N.B.—The repeater 9 is always considered equal to unity.

In the following examples reduce each quantity to a fraction and prove the equation.

Exercise 55.

(1) $\cdot\dot{4} + \cdot\dot{5}\dot{3} = \cdot 9\dot{7}\dot{9}$ (2) $\cdot 1\dot{6} + \cdot\dot{2}\dot{3} = \cdot 3\dot{9}\dot{8}$
(3) $1\cdot 5\dot{4} + 2\cdot 13\dot{2} = 3\cdot 67\dot{6}$ (4) $\cdot 0\dot{3} + \cdot 0\dot{7} = \cdot 10\dot{8}$

Addition, etc., of Circulating Decimals.

Rule.—Reduce each to a vulgar fraction and then add.

N.B.—In most cases it is sufficient to carry out the decimal quantities to as many places as may be required to ensure correctness, and then add as in common addition. Thus, add $3\cdot\dot{5} + 2\cdot 1\dot{7} + 3\cdot 6\dot{5} + \cdot 0\dot{1}$.

$$\begin{array}{r} 3\cdot 555555 \\ 2\cdot 171717 \\ 3\cdot 655555 \\ \cdot 010101 \\ \hline 9\cdot 392929 \end{array}$$ *Ans.*

The same plan may be followed in subtracting repeating decimal quantities.

In multiplying and dividing, each quantity must be reduced to a vulgar fraction.

Exercise 56.

(1) Add $1\cdot 35 + \cdot 03\dot{7} + 11\cdot 9 \times 5\cdot 7\dot{3}$; and add also $9\cdot 6\dot{8} + 2\cdot 2\dot{5}\dot{9} + 3\cdot 6 + 3\cdot 51$ correctly to nine places.

(2) Add $3\cdot 01 + 5\cdot 263\dot{4} + 18\cdot 29\dot{6} + 16\cdot 2\dot{5}$; and add also $4\cdot 1\dot{2} + 4\ 1523 + 16\cdot 07\dot{4} + 18\cdot 4\dot{7}$ correctly to ten places.

(3) From 9·01 take 7·12 ; also from 16·78 take 14·89.
(4) From 11·25 take 1·25 ; also from 18·36 take 8·36.
(5) Multiply 2·37 by 1·5 ; and 1·18 by 3·1.
(6) Multiply 37·9 by 25·13 ; and 18·9 by 50·26.
(7) Divide 3·72 by 1·7 ; and 11·16 by 5·3.
(8) Divide 9·001 by ·003$\dot{6}$; and 63·007 by ·025$\dot{4}$.

To reduce one quantity to the decimal of another.

Rule.—Take the quantity of the lowest denomination in the given quantity and divide by as many as make one of the next greater denomination to the quotient which will be a decimal, prefix the number of the next higher denomination in the quantity, and divide by the number of these which make one of the next higher ; continue this operation till the highest denomination named has been prefixed to the decimal, or until you have reduced it to the denomination sought.

Should the second quantity contain more than one denomination, reduce both quantities to the same name, and divide the second by the first.

EXAMPLE.—Reduce 3 qrs. 14lbs. to the decimal of a ton.

28)14·0
4)3·5
20)·875
·04375 *Ans.*

EXERCISE 57.

(1) Reduce 1 cwt. 1 qr. 21 lbs. to the decimal of a ton ; and 23 lbs. to the decimal of 2 cwt. 3 qrs. 12 lbs.
(2) Reduce 10*s.* 6*d.* to the decimal of £1 ; and 2*s.* 7½*d.* to the decimal of 5*s.* 0*d.*
(3) Reduce 13*s.* 11½*d.* to the decimal of 1 guinea ; and £2 8*s.* 10¼*d.* to the decimal of £3 13*s.* 6*d.*
(4) Reduce 5 oz. 10 dwts. 12 grs. to the decimal of 1 lb. Troy ; and 9 dwts. 5 grs. to the decimal of an ounce.

Reduce the first two quantities in each examples to

decimals, add them together, and prove by reducing the third quantity to a decimal.

(5) Reduce 15 lbs., 19 lbs., 1 qr. 6 lbs. to decimals of a cwt.

(6) Reduce 2 qrs. 10 lbs., 3 qrs. 19 lbs., and 1 cwt. 2 qrs. 1 lb. to decimals of a ton.

(7) Reduce 5*s.* 9*d.*, 4*s.* 6½*d.*, and 10*s.* 3½*d.* to decimals of £1.

(8) Reduce 3 oz. 10 dwts., 5 oz. 14 grs., and 8 oz. 10 dwts. 14 grs to the decimals of a lb. Troy.

(9) Reduce 3 yds. 2 ft., 7 fur. 15 poles 3 yds., 7 fur. 16 poles 1 yd 6 in. to the decimal of a mile.

(10) Reduce 15 hours 17 min., 13 hours 50 min., and 1 day 5 hours 7 mir. to the decimal of a week.

(11) Reduce 19 poles 27 yds., 27 poles 15½ yds., and 1 rood 7 poles 12¼ yds. to the decimal of an acre.

(12) Reduce £3 17*s.* 9½*d.* to the decimal of £11 16*s.* 8½*d.*; and 2 tons 10 cwt. 1 lb. to the decimal of 7 tons 12 cwt. 0 qrs. 19 lbs.

To find the value of the decimal of a given quantity, that is, to express a decimal in the known parts of the integer.

Rule.—Multiply the given decimal by the number of units of the next lower denomination which makes one of the given denomination—if there be an integral part in the product it will represent the number of units of that denomination—then reduce the decimal part to the next lower denomination, and proceed as before.

If the given quantity contain more than one denomination, it should be reduced to one before applying the above rule.

EXAMPLE 1.—What is the value of ·0576 of £1, or, as it may be expressed, £·0576 ?

·0576
20
1·1520
12
1·8240
4
¾·2960 *Ans.* 1*s.* 1¾*d.*

XAMPLE 2.—What is the value of ·075 of £5 9*s.* ?

£5 9*s.* = 109*s.*

·075

545

763

8·175

12

2·100

4

·400 *Ans.* 8*s.* $2\frac{1}{10}d.$

n the following examples find the values of the three ntities separately, the third will be equal to the sum of other two.

EXERCISE 58.

) £·285, £5·75, and £6·035.
) £11·9875, £13·645, and £25·6325.
) ·351 of a guinea, ·726 of a guinea, and 1·077 of a guinea.
) 3·7145 cwt., 2·516 qrs., and 4·34315 cwt.
) 3·71946 tons, 5·24678 tons, and 8·96624 tons.
) 5·376 yds., 4·982 yds., and 10·358 yds. (Cloth Measure)
) ·0563 miles, ·7987 miles, and ·855 miles.
) 24·7989 days, 13·5627 days, and 38·3616 days.
) 17·253 poles, 9·724 poles, and 26·977 poles.
0) 79·5186 cub. yds., 16·893 cub. yds., and 96·4116 cub. yds.

.B.—If the quantity contains a repeating decimal, carry out the es as far as will ensure correctness for the requisite number of es, or reduce the decimal to a vulgar fraction and then find its e.

reat these examples like the last.

1) £7·53, £6·36, and £13·29.
2) 3·175 cwt., 4·033 cwt., and 7·208508 cwt.
3) 7·6 miles, 5·89 miles, and 13·56 miles.

MISCELLANEOUS EXAMPLES IN DECIMALS.

EXERCISE 59.

) What will 39 books cost at ·375*s.* each? also 19·5 lbs. of sugar at per lb.

(2) Spent £·875 out of 3·75 guineas, how much is left? and find the value of £6·890625 divided by $2\frac{1}{4}$.

(3) Gained £17·35 and lost £5·386, how much did I really gain? also paid £8·497 out of £20·461.

(4) What will 13·45 yds. of cloth come to at £3·675 per yd.; also 40·425 yds. at £1·2227 per yd.?

(5) Gave £7·75 for 3·875 yds. of cloth, how much is that per yd.? also £1·55 for 3·1 qrs., how much is that per yd.?

(6) Add together $\frac{3}{8}$, $\frac{5}{7}$, and $\frac{9}{10}$ fractionally, and prove the operation decimally.

(7) From $3\frac{11}{40}$ take $1\frac{5}{12}$ fractionally, and prove decimally.

(8) Multiply 2 cwt. 3 qrs. 14 lbs. by 5·1435; also 5 cwt. 0 qrs. 16·072 lbs. by 2·875.

(9) Find the value of ·1796$\frac{1}{4}$ of a ton; also find the sum of 2 cwt. 3 qrs. 13·9875 lbs., and ·03589 ton.

(10) Reduce $\frac{375}{589}$ to a decimal; also reduce $\frac{25}{31}$ and $\frac{15}{19}$ to decimals, and find their product.

(11) Divide 7 oz. 10 dwts. 12 grs. by 1·3263; and divide 1 lb. 10 oz. 11 dwts. 12 grs. by 3·979.

(12) Add $\frac{3}{5}$, $4\frac{1}{3}$, $\frac{5}{8}$, and $\frac{7}{8}$ together fractionally and decimally.

(13) Divide £796 16*s.* 6*d.* by £3 18*s.* 9*d.* decimally, and prove by ordinary division of compound quantities.

(14) Bought ·38975 of a piece of cloth which measured 77·75 yds., find the value at 5*s.* $6\frac{3}{4}$*d.* per yd.; also bought ·194875 of a piece of cloth which measured 38·875 yds., find the value at £1·1125 per yd.

PRACTICE.

This peculiar name is given to an easy method of finding the value of any number of articles or any quantity of an article when the price of an integer or unit is given.

The principle upon which practice is worked is this, that the value of any part of a quantity at a certain price is the same portion of the whole value as the part taken is of the whole quantity, thus 1000 articles at 9*d.* come to 37*l.* 10*s.* and 1000 articles at 3*d.*, being one-third of the price, will be $\frac{1}{3}$ of the value, or 12*l.* 10*s.*

It will be seen that this is a short method of multiplication or proportion, and it is necessary in this rule to use aliquot parts. Aliquot means 'just so many'—that is 2 is

an aliquot part of 6, because 2 is contained in 6 just so many or rather an exact number of times.

To explain further—suppose the value is required of 739 at $10\frac{1}{2}d.$—it is evident that if the price were 1*s.* instead of $10\frac{1}{2}d.$, the answer would be 739*s.*; now it is also clear that $1d.$ being $\frac{1}{2}$ of a shilling, the value at $6d.$ will just be one-half the value at a shilling: the sum is then worked thus:—

EXAMPLE.

		s.		
6	$\frac{1}{2}$	739		
3	$\frac{1}{4}$	369	$6d.$ value at	$6d.$
$1\frac{1}{2}$	$\frac{1}{8}$	184	$9d.$ „	$3d.$
		92	$4\frac{1}{2}d.$ „	$1\frac{1}{2}d.$
		2,0)64.6	$7\frac{1}{2}d.$ „	$10\frac{1}{2}d.$
		£32	6*s.* $7\frac{1}{2}d.$	

Having found the value at $6d.$, the value at $3d.$ will be $\frac{1}{2}$ as much, I herefore divide it by 2; now, if I added these two lines together, it would give the value at $9d.$; take the aliquot part of 3 for the remaining $1\frac{1}{2}d.$, and divide; this will give the value at $1\frac{1}{2}d.$ Having thus found the values at $6d.$, $3d.$, and $1\frac{1}{2}d.$, I have merely to add them together for the value in shillings at $10\frac{1}{2}d.$, reduce to pounds, and the sum is done.

Before going on with the examples it is well to be acquainted with the aliquot parts of the most common units of quantity.

TABLE OF ALIQUOT PARTS.

Of a Ton		Of a Cwt.		Of a £.		Of a Shill.		Of a Lb.	
cwt.		lbs.		*s.* *d.*		d.		oz.	
10	$\frac{1}{2}$	56	$\frac{1}{2}$	10 0	$\frac{1}{2}$	6	$\frac{1}{2}$	8	$\frac{1}{2}$
5	$\frac{1}{4}$	28	$\frac{1}{4}$	6 8	$\frac{1}{3}$	4	$\frac{1}{3}$	4	$\frac{1}{4}$
4	$\frac{1}{5}$	16	$\frac{1}{7}$	5 0	$\frac{1}{4}$	3	$\frac{1}{4}$	2	$\frac{1}{8}$
$2\frac{1}{2}$	$\frac{1}{8}$	14	$\frac{1}{8}$	4 0	$\frac{1}{5}$	2	$\frac{1}{6}$	1	$\frac{1}{16}$
2	$\frac{1}{10}$	8	$\frac{1}{14}$	3 4	$\frac{1}{6}$	$1\frac{1}{2}$	$\frac{1}{8}$		
		7	$\frac{1}{16}$	2 6	$\frac{1}{8}$	1	$\frac{1}{12}$		
		4	$\frac{1}{28}$	2 0	$\frac{1}{10}$				
		$3\frac{1}{2}$	$\frac{1}{32}$	1 8	$\frac{1}{12}$				
		2	$\frac{1}{56}$	1 4	$\frac{1}{15}$				
		1	$\frac{1}{112}$	1 3	$\frac{1}{16}$				
				1 0	$\frac{1}{20}$				

(2) Spent £·875 out of 3·75 gv
value of £6·890625 divided by ?
(3) Gained £17·35 and lo
also paid £8·497 out of £20·
(4) What will 13·45 yds.
40·425 yds. at £1·2227 per
(5) Gave £7·75 for 3·8
also £1·55 for 3·1 qrs., h
(6) Add together
decimally.
(7) From 3
(8) Multiply 2
16·072 lbs. by 2·
(9) Find the
3 qrs. 13·9875
(10) Redu
and find the
(11) Di
10 oz. 11
(12)
(13
ordir

ner if
han
otie

· 0¾d.

ERCISE 60.

(2) 4738 at ¼d.
2369 „ ½d (3)

actice operations when the quantity is as
st remainder in every line will be of the same
given quantity.

(

CASE 2.—When the given price exceeds ¾d. and is
fir 1s, consider the quantity as shillings, take aliquot
e the price, the first part being taken out of a shilli
the quotients together, and the result reduced will
answer.

N.B.—If the price exceeds 1*s.* and is less than 2*s.*, add in the
with the several quotients, and reduce as before.

EXAMPLE 1.

4	½	2474 at 4¼
½	⅛	8248
		1031
		2,0)92,7 9
		£46 7*s.* 9*d.*

EXAMPLE 2.

6	½	3055 at 1*s.* 8¼*d.*
2	⅛	1527 6
¼		509 2
		63 7¾
		2,0)545.5 3¾
		£272 15*s.* 3¾*d.*

EXERCISE 61.

(1) 2796 at 0*s.* 1¼*d.*
(2) 932 „ 0*s.* 3¾*d.*
7866 „ 0*s.* 2½*d.*
874 „ 1*s.* 10½*d.*

(3) 3147 „ 0*s.* 5½*d.*
1049 „ 1*s.* 4½*d.*
(4) 7932 „ 0*s.* 7¼*d.*
2644 „ 1*s.* 9¾*d.*

0s. 11d.
1s. 4½d.
0s. 11¾d.
1s. 11¼
1s. 8
0s.

s of pounds, shillings, and
tiply by the pounds and
bring the pounds and
given quantity by the
the price, add the

1s. 3½
,, 0s. 7¾d.

e :— £ s. d.
79 at 7 13 2½

SE 3.—If the price is an e
ultiply by half the price and double
of the product for shillings, the rest of th
pounds.

EXAMPLE.

379 at 16s.
8
£303,4s. *Ans.*

EXERCISE 62.

(1)	2796	at	4s.	0d.	(4)	2352	at 10s. 0d.
	1398	,,	8s.	0d.		1680	,, 14s. 0d.
(2)	576	,,	6s.	0d.	(5)	2171	,, 18s. 0d.
	288	,,	12s.	0d.		6513	,, 6s. 0d.
(3)	2796	,,	8s.	0d.	(6)	3798	,, 12s. 0d.
	1398	,,	16s.	0d.		2532	,, 18s. 0d.

CASE 4.—When the price is an odd number of shillings only, multiply by the shillings and divide by 20.

EXAMPLE.

379 at 13s.
13
1137
379
2,0)492,7
£246 7s.

Exercise 63.

(1)	2331	at	7*s.* 0*d.*	(3)	323	at	19*s.* 0*d.*
	1813	„	9*s.* 0d.		361	„	17*s.* 0*d.*
(2)	995	„	15*s.* 0*d.*	(4)	351	„	11*s.* 0*d.*
	4975	„	3*s.* 0*d.*		297	„	13*s.* 0*d.*

Case 5.—When the price consists of shillings and pence above 2*s.*

There are two methods. Multiply the quantity by the shillings, and take parts of the pence out of the given quantity, and reduce as before, or take the aliquot parts of 1*l.* and proceed as before.

Example.

			372	at	3*s.*	8½*d.*		*s.*	*d.*			372	at	3*s.*	8½*d.*
6	½		3					3	4	⅙		62			
			1116						4	1/16		6	4		
2	⅓		186						½	⅛			15	6	
½	¼		62									£68	19*s.*	6*d.*	
			15	6											
		2,0)137,9	6												
		£68	19*s.*	6*d.*											

In the second example the first aliquot part being taken out of 1*l.*, it is evident the first remainder in every division will be pounds to be reduced and divided again. If the price be an aliquot part, divide by the proper divisor.

Exercise 64.

(1)	399	at	1*s.* 3*d.*	(7)	647	at	19*s.* 4¾*d.*
	133	„	3*s.* 9*d.*		4529	„	2*s.* 9¼*d.*
(2)	795	„	5*s.* 9*d.*	(8)	2739	„	4*s.* 7¾*d.*
	1590	„	2*s.* 10½*d.*		913	„	13*s.* 11¼*d.*
(3)	896	„	2*s.* 6*d.*	(9)	452	„	14*s.* 6*d.*
	128	„	17*s.* 6*d.*		904	„	7*s.* 3*d.*
(4)	3825	„	2*s.* 7⅓*d.*	(10)	3625	„	3*s.* 4*d.*
	765	„	13*s.* 1½*d.*		725	„	16*s.* 8*d.*
(5)	2328	„	3*s.* 11¼*d.*	(11)	1393	„	1*s.* 8*d.*
	582	„	15*s.* 9*d.*		199	„	11*s.* 8*d.*
(6)	1673	„	2*s.* 8½*d.*	(12)	824	„	16*s.* 8*d.*
	239	„	18*s.* 11½*d.*		6592	„	2*s.* 1*d.*

CASE 6.—If the price consists of pounds, shillings, and pence, there are two ways; multiply by the pounds and take parts of the first quantity—or bring the pounds and shillings into shillings, multiply the given quantity by the shillings, take parts for the rest of the price, add the quotients and divide by 20.

EXAMPLE.—

		£	s.	d.
10s. ½	279 at	7	13	2½
	7			
	1953			
2s. ⅛	139	10		
1s. ½	27	18		
2d. ⅙	13	19		
	2	6	6	
½d. ¼		11	7½	
	2137	5	1½	*Ans.*

The same :—

		£	s.	d.
2d. ⅙	279 at	7	13	2½
	153			
	837			
	1395			
	279			
	42687			
½d. ¼	46	6		
	11	7½		
20)4274,5		1½		
	2137,5	1½	*Ans.*	

EXERCISE 65.

		£	s.	d.
(1)	1794 at	1	16	7¾
	897 „	3	13	3½
(2)	3626 „	5	10	11½
	7252 „	2	11	5¾
(3)	4004 „	2	7	9½
	1001 „	9	11	2
(4)	2627 „	13	15	4½
	5254 „	6	17	8¼
(5)	4328 at	3	11	3
	2164 „	7	2	6
(6)	5795 „	1	11	2¾
	1159 „	7	16	1¾
(7)	2794 „	2	19	8¼
	254 „	32	16	6¾
(8)	6428 „	8	5	3¼
	3214 „	16	10	6½

CASE 7.—When the given quantity has a fraction annexed, find the value of the integral part of the quantity by previous cases, and before ascertaining the final result add in the value of the fractional part of the quantity, which is found by multiplying the price by the numerator and dividing by the denominator.

EXAMPLE.—Find the value of 323¾ at 15s. 9d.

	£	s.	d.	
10s. ½	323	0	0	
5s. ½	161	10	0	
6d. 1/20	80	15	0	
3d. ¼	8	1	6	
	4	0	9	
¾ of 15s. 9d. =	0	11	9¾	
	£254	19	0¾	Ans.

EXERCISE 66.

			£	s.	d.				£	s.	d.
(1)	379¼	at	0	0	9¾	(4)	796⅝	at	0	7	9½
	1137¾	„	0	0	3¼		1593⅜	„	0	3	10¾
(2)	510⅜	„	0	2	8½	(5)	7218¾	„	0	1	11½
	1020¾	„	0	1	4½		3609⅜	„	0	0	11¾
(3)	589⅘	„	0	1	8	(6)	989 7/16	„	5	11	9½
	294 9/10	„	0	0	10		494 23/32	„	11	3	7

CASE 8.—When the value of a compound quantity is required, multiply the price by the integral part of the quantity, and take parts for the rest, the sum of the quotients will be the answer.

EXAMPLE.—What is the value of 9 cwt. 1 qr. 7 lbs., at 3*l.* 10*s.* 6*d.* per cwt.?

	£	s.	d.
	3	10	6
1 qr. ¼			9
	31	14	6
7 lbs. ¼	0	17	7½
	0	4	4¾
	£32	16	6¼

In these sums the remainders from the farthings may be omitted by the learner on doing these sums for the first time; afterwards the exact answer to the fraction should be given.

Exercise 67.

Find the value of—

			£	s.	d.	
(1)	7 cwt. 2 qrs. 8 lbs.	at	1	19	8½	per cwt.
	3 cwt. 3 qrs. 4 lbs.	„	3	19	5	„
(2)	15 cwt. 1 qr. 7 lbs.	„	7	8	10½	„
	45 cwt. 3 qrs. 21 lbs.	„	2	9	7½	„
(3)	5 oz. 9 dwts. 11 grs.	„	7	10	8½	per oz.
	10 oz. 18 dwts. 22 grs.	„	3	15	4¼	„
(4)	17 qrs. 3 bus. 3 pks.	„	4	15	11½	per qr.
	34 qrs. 7 bus. 2 pks.	„	2	7	11¾	„
(5)	19 acres 3 rds. 17 poles	„	35	17	9½	per acre.
	39 acres 2 rds. 34 poles	„	17	18	10¾	„
(6)	7 yds. 3 qrs. 2 nls.	„	0	16	11¼	per yard.
	23 yds. 2 qrs. 2 nls.	„	0	5	7¾	„
(7)	3 cwt. 2 qrs. 25 lbs.	„	3	10	0	per ton.
	18 cwt. 2 qrs. 13 lbs.	„	0	14	0	„
(8)	10 oz. 3 dwts. 7 grs.	„	25	16	8	per lb.
	1 lb. 8 oz. 6 dwts. 14 grs.	„	12	18	4	„

The rule of practice may be usefully applied in many nstances of common occurrence, when the questions may be esolved into examples like those in the preceding cases.

It will be proper here to introduce a few examples relative o trade allowances.

Tare is an allowance for the weight of the box, bag, etc., nd is usually at so much for each package or per cwt.

Tret is an allowance, generally of 4 lbs. per 104 lbs., made for goods liable to waste, after tare has been deducted.

Cloff is an allowance of 2 lbs. in every 3 cwt. on certain articles.

Suttle is what remains after deducting the tare. After all deductions have been made, the remainder is called net weight.

Gross weight is the whole weight of the goods and package.

Example.

What is the net weight of 14 packages of tea weighing gross 7 cwt. 1 qr. 10 lbs., tare 10 lbs. per package ?

	cwt.	qrs.	lbs.	
	7	1	10	gross
lbs. 10 × 14 =	1	1	0	tare
	6	0	10	net weight.

When the tare is at so much per cwt. take aliquot parts as in Practice.

(9) Required the net weight of 25 packages of tobacco, each 2 qrs. 18 lbs.; tare 7 lbs. per package.

Find the net weight of 50 bags of sugar, each weighing 37 lbs. gross, tare per bag $3\frac{1}{2}$ lbs.

(10) From 162 cwt. 3 qrs. 14 lbs. gross, take tare 8 lbs. per cwt., and tret 4 lbs. per 104 lbs.

What tare must be allowed on 508 cwt. 0 qrs. $12\frac{1}{4}$ lbs., at 32 lbs. per cwt.?

(11) What is the net weight of 35 hhds. of sugar, each 15 cwt. 1 qr. 14 lbs. gross, tare per hhd. 18 lbs.?

Required the net weight of 105 casks of sugar, each 5 cwt. 14 lbs, tare per cask 6 lbs.

MISCELLANEOUS EXAMPLES IN PRACTICE.

Exercise 68.

(1) What is the value of 2791 yds. of cloth, at 15*s.* 9*d.* per yard; also of 5582 yards, at 7*s.* $10\frac{1}{2}d.$ per yard?

(2) Value 1725 bags of coffee, at 1*l.* 9*s.* $7\frac{1}{2}d.$ per bag; also, $431\frac{1}{4}$ ounces of gold, at 5*l.* 18*s.* 6*d.* per ounce.

(3) If a gentleman spends on an average of 1*l.* 8*s.* $5\frac{1}{4}d.$ per day, how much is that in a year? also value 2555 cwt. of salt, at 4*s.* $0\frac{3}{4}d.$ per cwt.?

(4) A dealer gains $3\frac{3}{4}d.$ in every shilling; what does he gain on 179*l.*; also value 716 books, at 1*s.* $6\frac{3}{4}d.$ each?

(5) Required the cost of 599 yards of fencing, at 7*s.* $7\frac{1}{4}d.$ per yd.; also $299\frac{1}{2}$ reams of paper, at 7*s.* $2\frac{1}{2}d.$ per ream.

(6) A tradesman fails for 2897*l.*, and can only pay 3*s.* $11\frac{1}{4}d.$ in the £; what is the value of his assets; also, value 11588 articles at $11\frac{13}{16}d.$ each?

N.B.—Assets mean his available property; this sum may be done by considering his debt as an abstract quantity, and 3*s.* $11\frac{1}{4}d.$ taken for every unit.

(7) A merchant paid 9*s.* $5\frac{1}{2}d.$ in the £ on a debt of 27969*l.* 17*s.* 6*d.*;

what did his creditors lose? also, find the value of 55939¾ yds. of velvet, at 4*s.* 8¾*d.* per yard.

(8) Find the weight of 1050 spoons, each 12 dwts. 16 grs.; also multiply 2 oz. 10 dwts. 16 grs. by 262½.

(9) What will the pay of 279 men for 6 months come to at 3*s.* 4¼*d.* per day; also find the value of 144 pieces of cloth, each 40½ yds., at 1*l.* 3*s.* 3*d.* per yard?

(10) What is the weight of 1376 books, each 1 lb. 10 oz. 10 drs.; also find the total weight of 426 parcels each 5 lbs. 6 oz.?

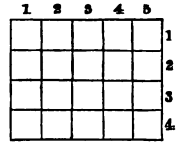
(11) Value 243 loads of hay, at 1*s.* 9½*d.* per truss, and 3 dozen pieces of cloth, each 21¾ yds., at 1*l.* 0*s.* 3*d.* per yard.

(12) A farmer pays rent on 243 a. 3 r. 10 p., at 3*l.* 10*s.* per acre; how much is that altogether? also, value 39970 yds. of ribbon, at 5¼*d.* per yard.

CALCULATION OF SURFACE, SOLIDITY, ETC.

The surface-content, or area of a right angled figure, may be found by multiplying the length and breadth together, and the number of cubic feet, &c., in a right angled solid body may be found by multiplying the three dimensions together, i.e. the length, breadth, and depth.

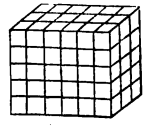
EXAMPLE 1.—Find the area of a figure whose length is 5 feet and breadth 4 feet. Multiply 5 by 4, and the product 20 is the answer.



This is evident if the figure be divided into parts each one foot square. The breadth in feet will represent so many rows, each containing as many square feet as there are feet in the length; therefore the breadth and length multiplied together gives the area.

EXAMPLE 2.—Find the solidity in feet of a block of stone—length 10 feet, thickness 7, breadth 5.

$10 \times 7 \times 5 = 350$ cub. ft. = 12 cub. yds. 26 ft.



If the whole figure is supposed to be divided into cubes of one foot each, there will be as many layers of them as there are feet in the depth or thickness, and each layer will contain as many as the product of the length and breadth. The whole then will be the length × breadth × thickness.

Note the following propositions.

1. The product of the length and breadth is the area or surface.

2. The product of length, breadth, and depth is the solid content.

3. To find the length, divide the area by the breadth.

4. To find the breadth, divide the area by the length.

5. If the solid content of a rectangular body be divided by one dimension, it will give the product of the other two; or if it be divided by two dimensions, it will give the third.

Let it be distinctly understood that one compound quantity cannot be multiplied by another compound quantity, because the latter as a multiplier cannot represent so many times. Therefore in multiplying feet and inches by feet and inches bring them to the same denomination, multiply and reduce the product by square or cubic measure.

EXAMPLE.—If the length of a room be 9 ft. 6 inches, and breadth 6 ft. 8 in.; how many square feet does it contain?

9 ft. 6 in. = 114
6 ft. 8 in. = 80

114
80

144)9120(63 ft. 48 in = $63\frac{1}{3}$ ft.
864

480
432

48

By Fractions:—

9 ft. 6 in. = $9\frac{1}{2}$
6 ft. 8 in. = $6\frac{2}{3}$

$\frac{19}{2} \times \frac{10}{3} = \frac{190}{3} = 63\frac{1}{3}$ feet.

By Decimals:—

$9{\cdot}5 \times 6{\cdot}6 = 63{\cdot}3 = 63\frac{1}{3}$ ft. *Ans.*

EXERCISE 69.

(1) Multiply 7 ft. 9 in. by 8 ft. 7 in.
also, 25 ft. 9 in. by 2 ft. 7 in.

(2) „ 12 ft. 10 in. by 3 ft. 10 in.
„ 6 ft. 5 in. by 7 ft. 8 in.

(3) Multiply 16 ft. 9 in. by 4 ft. 8 in.
„ 8 ft. $4\frac{1}{2}$ in. by 9 ft. 4 in.
(4) „ 27 ft. 11 in. by 26 ft. 3 in.
„ 83 ft. 9 in. by 8 ft. 9 ft.
(5) „ 19 ft. $8\frac{3}{8}$ in. by 17 ft. $9\frac{1}{4}$ in.
„ 39 ft. $4\frac{3}{4}$ in. by 8 ft. $10\frac{7}{8}$ in.

In this last example, reduce to inches and decimals.

(6) Multiply 279 ft. 7 in. by 326 ft. 7 in.
„ 559 ft. 2 in. by 163 ft. $3\frac{1}{2}$ in.
(7) Multiply together 3 ft. 7 in. × 5 ft. 9 in. and 6 ft. 11 in.
„ „ 1 ft. $9\frac{1}{2}$ in. × 2 ft. $10\frac{1}{2}$ in. and 27 ft. 8 in.
(8) „ „ 8 ft. 8 in. × 7 ft. 6 in. and 16 ft. 4 in.
„ „ 4 ft. 4 in. × 3 ft. 9 in. and 65 ft. 4 in.

Exercise 70.

(1) What is the area of a square yard whose side is 116 ft. 9 in.; also, the area of a garden measuring 233 ft. in by 58 ft. $4\frac{1}{2}$ in.

(2) How much carpet will be required for a room measuring 11 ft. 9 in. by 10 ft. 8 in.; also, find the surface of a wall 7 ft. 10 in. by 16 feet.

(3) On how many square yards of ground does a rectangular building stand whose length is 159 ft. 6 in., and front 78 ft. 10 in.? also, multiply 157 ft. 8 in. by 79 ft. 9 in.?

(4) How much paper 2 ft. 3 in. wide, will cover the walls of a room, whose sides are 10 ft. 10 in. by 11 ft. 6 in.?

How much floor-cloth 13 ft. 6 in. wide will cover a hall measuring 32 ft. 6 in. by 23 ft.?

(5) How many cubic feet of water may be contained in a tank measuring 15 ft. 9 in. in length, by 6 ft. 6 in. in height, and 9 ft. 6 in. in breadth?

How many cubic yards of earth were taken out of a hole measuring 63 ft. long, 3 ft. 3 in. wide, and 4 ft. 9 in. deep?

(6) Find the solid content of a mass of marble measuring 7 ft. 9 in. every way; also, of 8 blocks, each 3 ft. $10\frac{1}{2}$ in. square every way?

(7) The average depth of a pond is 5 ft. 9 in., width 45 ft. 8 in., and length 79 ft. 10 in.; how many cubic feet of water will it hold; also find the continued product of 22 ft. 10 in., 39 ft. 11 in. and 23 feet.

PROPORTION.

Proportion depends on a similarity of ratio. This may be remarked of numbers or quantities that depend on each other. To make this clear, observe that price depends on weight, size, or length; that is, as the weight, size, or length, so is the price. Again, time and wages, principal and interest, time and interest, are respectively proportional.

Ratio means the rate of proportion, in other words, it is the fraction which one quantity is of another, thus 6 to 8 has the ration of $\frac{6}{8}$ or $\frac{3}{4}$, and 8 to 6 the ratio of $\frac{8}{6}$ or $\frac{4}{3}$.

A simple proportion consists of two pairs of quantities having equal ratios, consequently there are 4 terms in a proportion when complete; thus, 3 is to 4 as 9 to 12, or dots may be used, thus, 3 : 4 :: 9 : 12; observe, the first term is the same fraction of the second that the third is of the fourth.

The outer numbers or quantities are called the extremes, and the inner two the means; and the product of the extremes is equal to the product of the means: thus, as above, $3 \times 12 = 4 \times 9$; consequently any three terms of a proportion being given, a fourth may be found, because the product of the two means divided by either extreme gives the other.

A proportion may consist of compound quantities; but to prove it, these compound quantities must be reduced to the same denomination, and considered as abstract numbers.

	lbs.	oz.		lbs.	oz.		*s.*	*d.*		*s.*	*d.*
EXAMPLE.—3	6	:	5	1	:	1	7	:	2	$4\frac{1}{2}$.	

By bringing the first and second to oz. and the third and fourth to halfpence, consider them as abstract numbers and prove the proportion.

	oz.		oz.		*s.*		*d.*
As,	54	:	81	::	19	:	$28\frac{1}{2}$
that is,	54	×	$28\frac{1}{2}$	=	81	×	19

Consequently from the above proportion, the following

question may be made. If 3 lbs. 6 ozs. of sugar cost 1*s.* 7*d.*, what will 5 lbs. 1 oz. cost?

Here it is evident that the price will increase in proportion to the weight, and consequently you have three terms of a proportion from which a fourth or the answer may be found.

State the question; that is, place the three terms given in proper order—to do this, consider of what denomination the answer will be—and make that quantity in the question which is of the same denomination the third term in the statement, and the quantity related to it must be placed as the first term, the remaining quantity being placed as the second term. Then reduce the terms if necessary, multiply the second and third together and divide by the first, the quotient will be the fourth term.

The above rule applies to questions in which the proportion is direct, that is *more* requiring more, or less requiring *less*, as in the instances before stated, viz. price and weight, time and interest; but there are cases in which more requires less, and *less, more*, as for example—principal and time, men employed and time—number of days and hours a day, &c., as it is evident the smaller the sum of money lent, the longer it must be at interest to gain a certain snm; or again, the more men you employ the less time they will require to perform a certain piece of work; or the more hours a day you travel the less number of days you will be occupied in a journey; these, and similar cases occurring frequently in practice, form what is called indirect or inverse proportion.

General Rule for Proportion.

Place that quantity which is of the same name as the answer for the third term, then consider whether the number sought will be greater or less than the third term: if greater, place the smaller of the other two quantities as the first term and the other as the second; if less, place the

greater of the other two quantities as the first term and the other as the second. Simplify the terms by reducing the first and second to the same name (the highest), and the third may be also reduced, to facilitate the operation; then multiply the third by the second and divide the product by the first, the quotient will be the fourth proportional or *answer* in the same denomination as that to which the third term was reduced.

Find a fourth proportional to 8, 12 and 15.

Here, $\frac{8}{12}$ gives the ratio that 15 must have with the fourth term; consequently,

$$\frac{12}{8} \times 15 = 22\tfrac{1}{2}$$

or multiply 12 by 15, and divide by 8 = $22\frac{1}{2}$.

Find a fourth proportional to 5, 20 and 16.

" " " 7, 28 :: 16.

" " " 9, 2, 27.

" " " $13\frac{1}{2}$, 3, 27.

EXAMPLE.—If 9 books cost 1*s.* 6*d.*, what will 13 books cost?

Here, observe the answer will be money, therefore money must be the third term; what you get for that money will be the first term, and the remaining quantity will be the second term. Thus,

books books *s.* *d.*

9 : 13 :: 1 6

$$\frac{13 \times 18}{9} = \frac{234}{9} = 26d. = 2s.\ 2d. \quad \textit{Ans.}$$

EXERCISE 71.

(1) If 7 yds. cost 1*s.* 9*d.*; what will 23 yds. cost?
" 14 " " $10\frac{1}{2}d.$ " " 92 yds. cost?

(2) At 5*s.* 3*d.* for 9 lbs.; what cost 89 lbs.?
" 3*s.* 6*d.* " 12 lbs.; " " 178 " ?

(3) At 17 yds. for 2*l.* 11*s.* 0*d.*; what cost 33 yds.?
" $25\frac{1}{2}$ " " 2*l.* 17*s.* $4\frac{1}{2}d.$ " " 44 " ?

(4) At 1*l.* 10*s.* 0*d.* per cwt.; what cost 3 qrs. 21 lbs.?
" 1*l.* 6*s.* 3*d.* " " " " 1 cwt. 0 qrs. 8 lbs.?

(5) Value 378 books, at 5*s.* 9*d.* per dozen.
" 672 " " 4*s.* $3\frac{3}{4}d.$ per 16.

(6) Value 189 yds., at 1*l.* 16*s.* for 12 yards.
" 336 " " 1*l.* 7*s.* for 16 yards.

(7) Value 24 reams, at 7*s.* 6*d.* for 8 quires.
„ 54 „ „ 5*s.* for 12 quires.

(8) Value 81 gallons, at 3*s.* 5*d.* for 6 gallons.
144 „ „ 2*s.* 6$\frac{3}{4}$*d.* for 8 gallons.

The process termed cancelling may be applied to the statement when the first and third or first and second terms have common factors; in such cases divide each pair of terms by such common factor after reducing them.

Thus, if 27 books cost 5*s.* what will 81 cost?

As, $\overset{1}{\cancel{27}}$: $\overset{3}{\cancel{81}}$: : 5

3 × 5 = 15*s.* *Ans.*

(9) If 8 pencils cost 1*s.* 8*d.*; what will 56 pencils cost?
„ 16 „ cost 5*d.*; what will 448 pencils cost?

(10) If 16 books cost 3*s.* 6*d.*; what will 108 cost?
„ 18 „ „ 5*s.* 3*d.*; „ „ 81 cost?

(11) If 25 yards cost 4*s.* 3*d.*; what will 175 cost?
„ 15 „ „ 7*s.* 1*d.*; „ „ 63 cost?

(12) At 3*s.* 6*d.* for 9 lbs.; what cost 126 lbs.?
„ 5*s.* 3*d.* „ 6 lbs.; „ „ 56 lbs.?

(13) At 14 for 1 guinea; how many for 2*l.* 14*s*?
„ 12 „ 24*s.* 6*d.*; „ „ „ 3*l.* 13*s.* 6*d.*?

(14) At the rate of 25 for 5*s.*; what will 385 cost?
„ „ „ 30 „ 4*s.* 2*d.*; „ „ 554$\frac{2}{5}$ cost?

Prove the following by Practice.

Required the value of—		£	*s.*	*d.*	
(15) 27 cwt. 2 qrs. 14 lbs.	at	3	16	6	per cwt.
(16) 17 oz. 10 dwts. 18 grs.	„	4	1	10$\frac{1}{2}$	per oz.
(17) 39 a. 4 r. 16 poles	„	3	16	4$\frac{1}{2}$	per acre.
(18) 59 qrs. 7 bus. 3 pks.	„	2	10	6	per quarter.
(19) 7 cwt. 27 lbs.	„	27	10	9	per ton.
(20) 15 poles 17 yards	„	3	12	8	per acre.

Get the exact answer to a fraction when proving the above 6 examples by Practice.

(21) If 2 cwt. 3 qrs. 10 lbs. of sugar cost 9*l.* 10*s.* 6*d.*; what will 11 cwt. 3 qrs. 12 lbs. cost?

If 4 cwt. 1 qr. 1 lb. cost 6*l.* 7*s.*; what is the value of 26 cwt. 2 qrs. 20 lbs. cost?

(22)) Gave 27*l.* 17*s.* 6*d.* for 50$\frac{5}{8}$ yds. of cloth; what will 120 yds. cost, at the same rate; also value 853$\frac{1}{3}$ yds. at the rate of 10*l.* 9*s.* 0$\frac{3}{4}$*d.* for 135 yds.?

(23) If 40 men can do a piece of work in 18 days, in what time will 160 men do the same?

If 18 men can mow a field in 9 days; how long will 36 men be doing the same?

(24) How much gold at 10*l.* 10*s.* 6*d.* for 2 oz. 6 dwts. can I buy for 175¾ guineas, and how much can I buy for 82*l.* 0*s.* 4*d.* at the rate of 3 oz. 9 dwts. for 7*l.* 0*s.* 4*d.*?

(25) If 130 three pound parcels can be made from a quantity of tea; how many 5 oz. packets can be made from the same; also at the rate of 5*s.* per lb., how many ounces can I buy for 19*l.* 10*s.*?

(26) What must I pay for 35¾ yds. of cloth, at 3*s.* 6*d.* for 1½ yd.?

Find the value of 11 cwt. 1 qr. 27 lbs. of soda at 9*s.* 4*d.* for 1 cwt. 1 qr. 4 lbs.?

(27) How much interest will be allowed on 796*l.* 10*s.*, at the rate of 5*l.* for 100*l.* or per cent., and how much on 531*l.* at 7½ per cent.?

(28) A bankrupt owes 793*l.*, and is only worth 132*l.* 3*s.* 4*d.*; how much can he pay in the pound? and another owes 1938*l.* 10*s.* 6*d.*, and pays 1615*l.* 8*s.* 9*d.*; how much will his creditors lose in the pound?

(29) How much is left of 290*l.* 10*s.* after paying for 130 sheep, at the rate of 17 for 25*l.* 18*s.* 6*d.*; and how much will pay for 90 sheep, at 11 for 11*l.* 5*s.* 6*d.*?

(30) Paid 7*s.* 4*d.* in the pound on 712*l.* 10*s.*; how much had I when I failed; and my neighbour failed and paid 7*s.* 6*d.* in the pound, which came to 97*l.* 19*s.* 4½*d.*; for what amount did he fail?

(31) How much in length that is 30⅔ yards broad will make an acre? and how much in breadth that is 460 yards long will make 15 acres?

COMPOUND PROPORTION.

This term is applied to questions involving more than one proportion statement, or containing more than three terms. A statement in compound proportion is but a combination of single proportion statements, and the question contains either 5, 7, 9, or any odd number of terms. There is, however, only one term that corresponds in denomination with the answer sought, and this is made the third term of the statement. The other terms are then taken in pairs, one

from the supposition part of the question, and its corresponding one in the demand part of the question; these terms being placed according to the rule for Single Proportion in reference to the direct or inverse nature of the answer. When all the single statements have been placed in order, it will be found that the third term is apparently common to all. Then multiply the first terms together, and all the second terms also, thus resolving all to one statement, and proceed as in Single Proportion.

EXAMPLE 1.—If 12 horses eat 16 bushels of oats in 8 days, how many bushels will 18 horses eat in 12 days?

Here the term corresponding with the answer is 16 bushels, this is put in the third term. 12 horses and 18 horses are then considered, then 8 days and 12 days; now the proportion is direct in both cases, because more horses more corn, and more days more bushels.

The statement then is—

hors.	hors.	bu.
12	18 ::	16
2	3 ::	2
8	12 ::	—
1	36 ::	—

Cancel numbers in the first and second, or first and third.

EXAMPLE 2.—If £100 gain £5 in 2 years, what will £350 gain in $3\frac{1}{2}$ years?

Here £5 is the third term, and the other terms taken in pairs are—

£100 : 350 (direct)
yrs. 2 : $3\frac{1}{2}$ (direct)

As £100 : 35,5 :: 5
2
2 $3\frac{1}{2}$
4 : $35 \times 3\frac{1}{2}$ :: 1

$\frac{35}{1} \times \frac{7}{2} \times \frac{1}{4} = \frac{245}{8} = £30\ 12s.\ 6d.$ *Ans.*

EXERCISE 72.

Each example contains a double question, and the answers considered as abstract numbers should be alike.

(1) If 6 men can dig a trench 20 yards long in 4 days, how long would 9 men require to dig a trench 15 yards long? also, under the same conditions, how many men would dig 25 yards in 15 days?

(2) If 20 horses eat 50 bushels of oats in 15 days, how much will 15 horses eat in 8 days ? and how many horses will eat 70 bushels in 21 days ?

(3) If 25 men can earn £170 in 8 weeks ; how much will 78 men earn in 10 weeks ?

If 37½ pieces of cloth, each 16 yards, cost £510, what will 39 pieces, each containing 20 yards, cost ?

(4) 16 persons spend £44 in 4 months, how much, at the same rate, will 40 persons spend in 10 months ?

If 18 horses eat 110 bushels in 8 weeks, how many bushels will 80 horses eat in 4½ weeks ?

(5) If 10 compositors can set up 560 pages in 7 days, how long will 4 men be in setting up 256 pages ? and how many dozen pages will 3 men set up in 4 days ?

(6) If £270 in 5 years gain £13 10*s.*, what will £150 gain in 15 years ? and, at the same rate, what will £562 10*s.* gain in 4 years ?

INTEREST.

Interest used commercially means profit derived from lending or investing money, and is usually reckoned at so much per cent. or per 100*l.*

The money lent is called the Principal.

What is paid for the loan is Interest.

What is paid for 100*l.* for one year is the Rate per cent. per annum.

The Amount is the principal and interest added together.

When the interest of a certain sum of money is required, the rate per cent. and time are given ; and the interest of any sum of money however large is proportionate to the rate per cent.

To find the Interest for one year.

Rule.—Multiply the principal by the rate per cent. and divide by 100, or take aliquot parts for the rate per cent., and the result will be the interest.

Example.—What is the interest of £286 15*s.* for 1 year at 5 per cent.?

£	*s.*
286	15
	5
14	33 15
	20
	6,75
	12
	9,00

or

5 = $\frac{1}{20}$ of £100

£	*s.*	*d.*
286	15	0
14	6	9

Exercise 73.

In the following exercise add the first two results in each example and the sum should be equal to the third result. Complete the results in each example.

(1) What is the interest of £395 for 1 year at 2, 3, and 5 per cent. ?

(2) What is the interest of £796 10*s.* 8*d.* at 2½, 3½, and £5 15*s.* per cent. ?

(3) What is the interest of £743 16*s.* at 4½, 3½, and 8 per cent. ?

(4) Required the interest of 917 15*s.* at 2⅜, 3¾, 6⅛ per cent.

(5) How much will £1,050 gain in 1 year at 5, 6, and 11 per cent. ?

When the Interest is for any number of years.

Rule.—Find the interest for one year and multiply it by the number of years, or multiply the principal by the rate per cent. and time, and divide by 100.

Or, take aliquot parts of the rate per cent. and time, and work as in Practice.

Example.—Interest is required of £350 for 3 years at 5 per cent.

£
350
3
1050
5
52,50 = £52 10*s.*

or

5 = $\frac{1}{20}$

£
350
17 10
3
£52 10*s.* *Ans.*

Exercise 74.

In the following exercise the result of the third question in each example should be equal to the sum of the results

of the first and second, each example being complete worked.

Find the interest of—

	£	s.	d.						
(1)	275	0	0	for	2	years	at	3	per cent.
				„	3	„	„	5	„
				„	4	„	„	$5\frac{1}{4}$	„
(2)	816	0	0	„	6	„	„	7	„
				„	7	„	„	4	„
				„	10	„	„	7	„
(3)	386	0	0	„	$4\frac{1}{2}$	„	„	5	„
				„	$3\frac{3}{4}$	„	„	4	„
				„	5	„	„	$7\frac{1}{2}$	„
(4)	426	10	6	„	3	„	„	4	„
				„	4	„	„	5	„
				„	10	„	„	£3 4*s.*	per cent.
(5)	796	15	0	„	$5\frac{3}{4}$	„	„	5	per cent.
				„	$17\frac{1}{2}$	„	„	6	„
	1593	10	0	„	10	„	„	£6 13*s.* 9*d.*	per cent.
(6)	2796	0	0	„	17	„	„	$2\frac{3}{5}$	per cent.
				„	20	„	„	$3\frac{1}{4}$	„
	13980	0	0	„	3	„	„	£7 0*s.* 6*d.*	per cent.
(7)	195	0	0	„	10	„	„	5	per cent.
				„	15	„	„	6	„
				„	35	„	„	4	„

To find the Interest for days.

Find the interest for one year, and then for the giv number of days by Proportion.

Thus, to find the interest of 150*l.* for 29 days at 5] cent.

The interest for 1 year is £7 10*s.*

	dys.	dys.	£	s.	d.
Then as	365	: 29 ::	7	10	0

This proportion completed gives the interest for 29 da

Exercise 75.

What is the interest of—

	£	s.	d.						
(1)	375	0	0	for	29	days	at	5	per cent. ?
	93	15	0	„	87	„	„	$6\frac{2}{3}$	„

£	s.	d.						
526	10	0	for	35	days	at	$4\frac{1}{2}$	per cent.
105	6	0	„	105	„	„	$7\frac{1}{2}$	„
206	6	4	„	120	„	„	4	„
154	14	9	„	128	„	„	5	„
750	10	0	„	250	„	„	3	„
600	8	0	„	125	„	„	$7\frac{1}{2}$	„

To find the Interest for years and days.

—Find the interest for one year, and multiply by ıber of years ; to this product add the interest for ², found by the last rule, and it will give the answer.

Exercise 76.

hat is the interest of £375 10*s.* for 2 years 119 days ? and of . 6*d.* for 3 years 37 days at 5 per cent. ?

ıat is the interest at 4 per cent. of £730 16*s.* 8*d.* for 3 years ıys ? also for £5115 16*s.* 8*d.* for 163 days ?

quired the interest at $3\frac{1}{2}$ per cent. of £374 16*s.* 8*d.* for 9 years ? also of £1874 3*s.* 4*d.* for 1 year 317 days.

he Rate, Amount or Interest, and Time are given, to find the Principal.

—As the amount of 100*l.* for the rate and time given)*l.*, so is the given amount to the answer.

LE.—What principal will amount to £250 in 4 four years at 5 ?

Amount of £100 for rate and time given = £120.

	£		£	£
As	12,0	:	10,0	:: 250
				10
				12)2500
				£208 6*s.* 8*d.* *Ans.*

Exercise 77.

ıat principal will amount to £325 in 5 years at 6 per cent. ?

principal will amount to £296 17*s.* 6*d.* in $3\frac{3}{4}$ years at 5 per

(2) What principal will gain £175 in 5 years at 5 per cent.? also what principal will gain £182 7*s.* in 5½ years, at 5½ per cent.?

When the Principal, Amount, or Interest are given, to find the Time.

Rule.—Subtract the principal from the amount, and it will give the whole interest; then as one year's interest of the principal is to the whole interest, so is one year to the answer.

EXAMPLE.—In what time will £320 amount to £368 at 5 per cent.?

Here interest of £320 for 1 year = £16;
and whole interest is £368 − 320 = £48.

£	£	
As 16	: 48	:: 1

16)48

3 years.

(3) In what time will £475 amount to £546 5*s.* at 3 per cent.? also in what time will £295 amount to £405 12*s.* 6*d.* at 7½ per cent.?

(4) In what time will £365 amount to £447 2*s.* 6*d.* at 5 per cent.? and in what time will £225 amount to £326 5*s.* at 10 per cent.?

When the Principal, Time, and Amount or Interest are given, to find the Rate per cent.

Rule.—Divide the whole interest by the interest of the principal at 1 per cent., the quotient will be the rate per cent.

EXAMPLE.—At what rate per cent. will £150 amount to £250 in 5 years?

Whole interest, £100.
Interest at 1 per cent., £7 10*s.*

£ *s.*	£	£	
As 7 10	: 100	:: 1	

2 2

15 15)200

$13\frac{1}{3}$ per cent.

(5) At what rate per cent. will £225 amount to £270 in 4 years? also at what rate per cent. will £392 10*s.* gain £70 8*s.* in the same time?

(6) At what rate per cent. will £1,000 gain £175 in 7 years? also at what rate per cent. will £295 amount to £317 2*s.* 6*d.* in 3 years?

INSURANCE COMMISSION, BROKERAGE, ETC.

Insurance is so much per cent. paid for security against loss from fire and other causes.

Commission is so much per cent. paid to any one for buying and selling goods.

Brokerage is a percentage paid to a broker for transacting pecuniary affairs, such as buying and selling stock, shares, &c.

Discount is so much deducted for ready money payment; it is generally calculated at so much per cent. for one year.

Rule.— Calculate as in simple interest or by taking aliquot parts.

EXERCISE 78.

(1) What premium must be paid for insuring goods valued at £765 at 2½ per cent.? also on £2295 at 16*s.* 8*d.* per cent.?

(2) What is the brokerage on a transaction of £5268 at 3*s.* 6*d.* per cent.? also on £7902 at 2*s.* 4*d.* per cent.?

(3) What discount will a tradesman allow on £125 15*s.* at 1*s.* on the £ or 5 per cent.? also on £167 13*s.* 4*d.* at 3¾ per cent.?

(4) Required the premium on a policy of insurance for £987 15*s.* at the rate of £2 1*s.* 8*d.*; also on £1975 10*s.* at £1 0*s.* 10*d.*

(5) What must be paid for insuring a vessel and cargo valued at £5796 15*s.* at 2¾ per cent.? also another valued at £11,593 10*s.* at £1 7*s.* 6*d.*?

(6) A broker sells shares to the amount of £20,189 10*s.*, what must he receive at 3*s.* 4*d.* per cent., and what is the brokerage on a transaction of £13,459 13*s.* 4*d.* at 5*s.* per cent.?

(7) What is the brokerage on £2198 10*s.* 6*d.* at 4*s.* 6*d.* per cent.? and on £4397 1*s.* at 2*s.* 3*d.* per cent.?

(8) How much discount should be allowed on £275 15*s.* at 2½ per cent.? also on £413 12*s.* 6*d.* at 1⅔ per cent.?

(9) What will be the premium on a policy of insurance for £796 10*s.* at £3 1*s.* 6*d.* per cent.? also on £1194 15*s.* at £2 1*s.* per cent.?

(10) Required the discount on £76 15*s.* at 5 per cent.; and on £153 10*s.* at 2½ per cent.

(11) How much commission is due to a merchant for selling goods to the value of £7938 16*s.* 3*d.* at 4*s.* 8*d.* per cent.? also find the premium of insurance on £21170 3*s.* 4*d.* at 1*s.* 9*d.* per cent.?

STOCKS.

This name is given to sums of money lent to a company or Government. The rate of interest and the price of the stock are fixed by the parties proposing the loan.

It must be understood that, although 100*l.* of scrip always bears its full interest, its buying and selling value varies according to circumstances affecting at the time the stock market and the general state of affairs. This variation in the price of stock causing its transfer or sale gives employment to stockbrokers, who are the agents for carrying on the stock business of the country.

Shares are at par when they fetch their nominal value.

They are below par or at a premium according as they are less or more than their nominal value.

Further information on stocks may be best given in explanation of some questions.

EXAMPLE 1.—If £2000 is invested in the 4 per cents. at 75; what interest will it bring?

By the 4 per cents. at 75 is meant that £100 stock in the 4 per cents. may be bought for £75 cash.

	£ cash.	£	£
Therefore	75 :	2000 ::	100
	3	4	4
		3)8000	

£2666 13*s.* 4*d.*—value of stock that can be purchased for £2000 at the rate of £75 for £100 stock bearing interest at 4 per cent.

Interest of £2666 13*s.* 4*d.* at 4 per cent. is £106 13*s.* 4*d.* *Ans.*

EXAMPLE 2.—What must I lay out in the $3\frac{1}{2}$ per cents. at 95 to produce an income of £150?

Here it is evident that $\frac{150}{3\frac{1}{2}}$ will give the number of cents. of stock on which interest is to be paid, and as each £100 of scrip is only worth £95 the price of the whole will be—

$$\frac{150}{3\frac{1}{2}} \times 95 = £4071\tfrac{3}{7} \text{ } Ans.$$

EXAMPLE 3.—What is the difference in income derived from £5000 in the 3 per cents. at 80, and the same sum in the 4 per cents. at 75?

Here $\frac{5000}{80} \times 3 =$ £187 10*s.* 0*d.* income derived from 3 per cents.;

and $\frac{5000}{75} \times 4 =$ £266 13*s.* 4*d.* income derived from 4 per cents.

£79 3*s.* 4*d.* in favour of 4 per cents.

EXAMPLE 4.—If I transfer £10000 from the 5 per cents. at 90 to the per cents. at 75, what difference in income will there be ?

As £10000 in the 5 per cents. at 90 is only worth £9000, if this be ıvested in the 3 per cents. at 75 it will purchase $\frac{9000 \times 100}{75} =$ £12,000 orth of stock.

£10,000 at 5 per cent.	produces	£500
£12,000 at 3 „	„	£360
		£140

There is a loss of £140 by the second investment.

EXERCISE 79.

(1) The 5 per cents. are at 60, what income shall I get by laying out 950 in that stock ? and the 4 per cents. being at 48, what income may) derived from a similar investment ?

(2) What income may be derived from £2500 invested in the 5 per ;nts. at 95 ? and the same sum laid out in 3½ per cents. at 66½ ?

(3) Which would produce the larger income, £2500 in the 3 per ;nts., or £1875 in the 4 per cents. ?

(4) How much must be laid out at 60 in the 4 per cents. to produce ı income of £100? and how much in the 3 per cents. at 45 to produce ıe same ?

(5) If a person lay out £3650 in the 5 per cents. at 91¼, how much ill he lose if he sells out at 88¾ ? and if the same sum be invested in ıc 4 per cents. at 73, how much would he gain if he sold out at 75 ?

(6) What will £755 purchase in the 3 per cents. at 75 ? and £563 ıs. 8*d.* in the 4 per cents. at 56 ?

(7) What income would £896 15*s.* 6*d.* produce if invested in the per cents. at 80 ? and the same sum laid out in 3½ per cents. at 70 ?

(8) A person invests £5050 in the 4 per cents. at 86, and another ys out the same amount in 3½ per cents. at 75¼, what are their re- ıective incomes ?

(9) If I lay out £2250 in the 4 per cents. at 75 and sell out at 76½, hat do I gain ? and what would £1012 10*s.* invested in 4 per cents. ; 90 produce ?

(10) What income would be derived from laying out £7872 in the 3 ır cents. at 82 ? and from £5040 laid out in 4 per cents. at 70 ?

COMPOUND INTEREST.

This differs from Simple Interest inasmuch as the in is calculated on principal and interest, each year's in being added to the corresponding principal, and the in calculated thereon for the next year.

Rule.—Find the first year's interest, which add t principal; then find the interest of this sum at the sam and add the interest to the principal, &c., for the number of years—the last sum will be the amount, which take the principal for the interest required.

EXAMPLE.—Find the compound interest of £50 for 3 years a cent.

		£	s.	d.	
5 $\frac{1}{20}$		50	0	0	
		2	10	0	first year's interest.
5 $\frac{1}{20}$		52	10	0	
		2	12	6	second year's interest.
5 $\frac{1}{20}$		55	2	6	
		2	15	$1\frac{1}{2}$	third year's interest.
		57	17	$7\frac{1}{2}$	
		50	0	0	
		£7	17	$7\frac{1}{2}$	*Ans.*

Prove these sums by taking different aliquot parts (rate per cent., or by working them decimally.

EXERCISE 80.

What is the compound interest of—

	£	s.	d.						
(1)	250	0	0	for	3	years at	5	per cent. ?	
(2)	570	0	0	„	4	„	6	„	
(3)	764	10	0	„	5	„	$4\frac{1}{2}$	„	
(4)	587	5	0	„	$4\frac{3}{4}$	„	4	„	

What would be the amount of—

	£	s.	d.						
(5)	300	0	0	in	$5\frac{1}{2}$	years at	7	per cent. ?	
(6)	1050	0	0	„	10	„	8	„	
(7)	325	0	0	„	4 years	7 mon.	10 dys.	at 5	per
(8)	560	0	0	„	7 „	8 „	15	„	$7\frac{1}{2}$

DISCOUNT.

This is the sum allowed for paying money before it is due.

Many tradesmen allow discount for ready-money payments, a per centage from the amount of the bill. The deduction is, however, only the interest of the amount, and not the true discount.

True Discount is the difference between the Present Worth and the amount of the bill.

The Present Worth of a bill not yet due is such a sum as would amount to the bill or debt on the day on which it is due, at any agreed rate per cent.

EXAMPLE.—What is the present worth of £250 due in 3 months at 5 per cent.? Here you want to find such a sum as would amount to £250 in 3 months on 5 per cent.

This is done by a case in interest.

As the amount of £100 for the rate and time given is to £100, so is the sum given to the present worth.

$$\text{As } \underset{\pounds}{101\tfrac{1}{4}} : \underset{\pounds}{100} :: \underset{\pounds}{250}$$

$$\frac{250 \times 100 \times 4}{405} = \frac{20000}{81} = £246\tfrac{74}{81}.$$

It must be noticed here, however, that bankers and others calculate as discount the interest on the debt itself, thus taking more than the true discount.

On a bill of £250 due in three months at 5 per cent., a banker would charge as discount £3 2*s.* 6*d.*, reducing the bill to £246 17*s.* 6*d.*; but. according to the last example, the present worth is £246 18*s.* $3\frac{1}{4}\frac{3}{81}d.$, the banker thereby gaining $9\frac{1}{4}d.$ by his method of calculating the discount.

To find the Present Worth and Discount.

Rule.—As the amount of £100 for the rate and time given : £100 :: the given sum to the present worth. From

the given sum take the present worth, it will give the discount; or to find the discount, as the amount of £100 is to the interest of £100 for the rate and time given, so is the given sum to the discount.

Prove these sums by ascertaining whether the present worth, as you have found it, would really amount to the debt in the given time.

What is the present worth of £250 due in 6 months at 5 per cent. ?

$$\text{As } 102\tfrac{1}{2} : 100 :: 250$$

$$\frac{\overset{50}{\cancel{250}} \times 100 \times 2}{\underset{41}{\cancel{205}}} = \frac{10000}{41} = £243\tfrac{37}{41} \text{ present worth.}$$

Now, say what will £$243\frac{37}{41}$ amount to in 6 months at 5 per cent. ?

$$\frac{\overset{250}{\cancel{10000}}}{41} \times \frac{1}{\underset{20}{\cancel{40}}} \times \frac{1}{2} = \frac{250}{41} = 6\tfrac{4}{41} \text{ Interest.}$$

$$243\tfrac{37}{41} + 6\tfrac{4}{41} = £250 \text{ proof.}$$

If the discount is first found, subtract from the given sum for the present worth, and prove as above shown.

Exercise 81.

(1) What is the present worth of £180 due in 1 year at 5 per cent. ?

(2) Required the discount on £250 due at the end of $2\frac{1}{2}$ years at 4 per cent.

(3) Find the present worth and discount of £365 16*s.* 8*d.* due in 9 months at 5 per cent.

(4) What is the present value of a bill for £500 due in 12 months at 6 per cent. ?

Find the discount on the following bills—

	£	*s.*	*d.*		Drawn.	Discounted.
(5)	375	10	0	May	3, at 3 months,	July 15, at 5 per cent.
(6)	420	15	0	April	12, at 6 „	Sept. 10, at 4 „
(7)	500	10	0	June	10, at 9 „	Nov. 15, at 5 „

BARTER

Is the exchanging of one commodity for another, and this rule shows how the exchange may be effected so that neither party may sustain loss.

In questions of this kind, the quantity and value of one party's goods are generally known; and the rate of value of the other's goods being known, it is easy to find an equivalent quantity.

For instance, A has 20 yards of cloth at 15*s.* 6*d.* which he wishes to exchange for paper with B; now it is evident that the rate of value of the paper must be known, to enable you to find how much paper must be given for the cloth; now let the paper be at 6*d.* per quire, then the number of quires will be equal to the number of sixpences in the value of the cloth.

Value of cloth 15*s.* 6*d.* × 20 = 620 sixpences.
620 quires of paper equal in value to the cloth.

Exercise 82.

(1) How much sugar at 6*d.* must be given for 3 qrs. 10 lbs. of tea, at 3*s.* 6*d.* per lb.?

What quantity of tobacco at 2*s.* 4½*d.* per lb. is equal in value to 1 cwt. 2 qrs. 20 lbs. of spice at 8*s.* 3¾*d.* per lb.?

(2) How much calico at 1*s.* 2*d.* per yard in addition to 100*l.* ready money must be given for 57 pieces of cloth each 20 yds. at 3*s.* 8*d.* per yard?

How many yards of ribbon at 3½*d.* per yard, must be given for 3 pieces of cloth, each 54½ yds. at 3*s.* 4*d.* per yard?

(3) If A gives B 1 cwt. of soap for 84 lbs. of coffee valued at 4*l.* 4*s.* per cwt., at what rate does A value his soap per lb.?

If B gives A 3 cwt. of coffee for 1 cwt. 2 qrs. 21 lbs. of butter valued at 5*l.* 12*s.* per cwt., at what rate per lb. has he reckoned his coffee?

(4) How much ready money must be given in addition to 2 cwt. 3 qrs. 14 lbs. of tea at 10*l.* 10*s.* 6*d.* per cwt. for 17 cwt. 2 qrs. 10 lbs. of sugar at 2*l.* 3*s.* 9*d.* per cwt.?

What is the difference in value between 1 cwt. 1 qr. 21 lbs. of tea at 15*l.* 15*s.* 9*d.* and 8 cwt. 3 qrs. 5 lbs. of sugar at 4*l.* 7*s.* 6*d.* per cwt.

(5) How many dozen of port at 37*s.* 6*d.* must be given for 28 dozen of sherry at 27*s.*?

How many dozen of champagne at 56*s.* 3*d.* for 21 dozen of brandy at 54*s.*?

(6) How many gross of pens at 2*s.* 6*d.* per gross must be given for 30 reams of paper at 9*d.* per quire?

How many ounces of silver at 6*s.* 10½*d.* are equal in value to 11 ounces of gold at 5*l.* 12*s.* 6*d* per ounce?

(7) Gave 15 cwt. 2 qrs. 7 lbs. of sugar at 4*l.* 10*s.* per cwt. and 100*l.* for 9 cwt. 12 qrs. 7 lbs. of tea; what was the tea valued at per cwt.?

How much ready money in addition to 10 cwt. of rice at 18*s.* 2*d.* per cwt. must be given for 18½ cwt. of soda at 1*l.* 10*s.* per cwt.?

(8) How much wheat at 45*s.* 6*d.* per quarter, for 70 qrs. oats at 21*s.* 6*d.* per qr.?

How much cloth at 3*s.* 9½*d.* per yd. must be given for 17½ yds. at 7*s.* 2*d.*?

EQUATION OF PAYMENTS.

This rule enables us to find a mean time for the payment of a sum equivalent to several sums due at different periods.

Rule.—Multiply each sum by the time it has to run and divide the sum of the products by the whole debt, the quotient is the mean equated time.

EXAMPLE.

Find a mean time for the payments of the following—30*l.* in 3 months, 40*l.* in 6 months, 60*l.* in 9 months?

$$\begin{array}{rcrcr} 30 & \times & 3 & = & 90 \\ 40 & \times & 6 & = & 240 \\ 60 & \times & 9 & = & 540 \\ \hline 13{,}0 & & & & 13)87{,}0 \end{array}$$

$6\frac{9}{13}$ months.

EXERCISE 83.

(1) A owes B 100*l.* in 6 months, 150 in 9 months, 250 in 12 months; *find a mean time* for one payment.

A merchant owes 500*l.* to be paid as follows:—40*l.* in 3 months, *l.* at 5 months, 60*l.* in 8 months, 200*l.* in 10 months, 150*l.* in 14 months; id a mean time for paying the whole sum?

(2) A tradesman owes 650*l.* payable as follows: 150*l.* in 6 months, 0*l.* in 8 months, and 300*l.* in 10 months; find a mean time for paying e whole debt.

A debt of 650*l.* is to be paid as follows: 150*l.* in 4 months, 80*l.* in months, 270*l.* in 10 months, and 150*l.* in 12 months; find a mean ne for paying the whole.

PROFIT AND LOSS.

By this rule is calculated the loss or gain in the purchase sale of articles.

Loss or gain is expressed at so much per cent.

Explanation is given with the following examples.

EXAMPLE 1.—Bought sugar at 35*s.* per cwt. and sold at 47*s.*; what d I gain on 11 tons?

Here $47 - 35 = 12s.$ gain per cwt.?

Also $12s. \times 220\text{cwt.} = 132l.$ *Ans.*

EXAMPLE 2.—If coffee be bought at 3*l.* per cwt. and sold at 4*l.* 5*s.*; hat is the gain per cent.?

Take the prime cost from the selling price, and it will give the ain—

$4l.\ 5s. - 3l. = 1l.\ 5s.$ gain on 3*l.*

en to find the gain per cent. or on 100*l.* we have the following pro- ortion:—

	£		£		£	s.
As	3	:	100	::	1	5

$1\frac{1}{4}$

100

25

3)125

£41 13*s.* 4*d.*

EXAMPLE 3.—Gained 10 per cent. by selling cloth at 15*s.* 7*d.* per ard; what was the prime cost?

Here, 100*l.* of goods must be sold for 110*l.* to gain 10*l.* per cent.; hence the proportion—

	£	s.	d.		
As 110 :	0	15	7	::	100
11			10		10

11)7 15 10

0 14 2 *Ans.*

EXAMPLE 4.—Bought paper at 1*l.* 10*s.* 6*d.* a ream and was compelled to sell at a loss of $5\frac{3}{4}$ per cent.; at what price did I sell it per ream?

Here, 100*l.* worth of goods must be sold for $94\frac{1}{4}$*l.* to lose $5\frac{3}{4}$ per cent.; therefore,

£		£	s.	d.		£
As 100	:	1	10	6	::	$94\frac{1}{4}$

$\frac{61}{2} \times \frac{377}{4} \times \frac{1}{100} = 1l.\ 8s.\ 8\frac{3}{4}d.$ $\frac{41}{80}$ *Ans.*

EXAMPLE 5.—If 17 per cent. be gained by selling corn at 45*s.* per qr.; shall I gain or lose by selling it at 35*s.*?

Here, selling at the rate of 45*s.* will enable you to get 117*l.* for 100*l.*

Therefore, as 45*s.* : 35*s.* :: 117

9 7 7

9)819

91 per cent.

Therefore, selling at the rate of 35*s.* will only enable you to get 91*l.* for 100*l.*; the consequence is you will lose 9 per cent.

EXAMPLE 6.—If 6 per cent. be lost by selling tea at 3*s.* 11*d.* per lb.; at what rate must I sell it to gain 10 per cent.?

Here, selling at 3*s.* 11*d.* only enables you to get 94*l.* for 100*l.*; but you want to get 110*l.* for 100*l.*

	£		s.	d.
As 94 :	110	::	3	11
2			12	
			47	

2)110 1

55*d.* = 4*s.* 7*d.* *Ans.*

EXERCISE 84.

(1) Bought linen at 4*s.* 8*d.* and sold it at 5*s.* 3*d.*; what was the gain per cent.; also, bought cloth at 12*s.* and sold it at 10*s.* 6*d.*; what did I *lost per cent.?*

(2) Sold tobacco at 10*l.* 9*s.* and lost thereby 5 per cent.; what was
e prime cost? and sold a piece of cloth for 12*l.* 13*s.* gaining thereby
per cent.; what was the prime cost?
(3) Lost 19 per cent. by selling at 27*s.*; what should I have gained
selling at 35*s.*?
Gained 17 per cent. by selling at 39*s.*: whether should I have gained
lost by selling at 35*s.*?
(4) Bought cloth at 3*s.* 6½*d.* and sold it at 4*s.* 11*d.*; how much did I
in by selling 250 yds. at that rate?
If I sell at 2*s.* 5½*d.* tea that I bought at 1*s.* 9¼*d.* per lb.; what shall
ain by selling 4 cwt. 1 qr. 24 lbs.?
(5) Bought linen at 2*s.* 3½*d.* per yard and sold 200 yards for 25*l.*;
at did I gain per cent.?
Sold cloth bought at 4*s.* 1½*d.* for 3*s.* 9*d.*; what did I lose per cent.?
(6) If hops are bought at 4*l.* 10*s.* 6*d.* per cwt. and sold for 5*l.* 1*s.* 8*d.*;
at is the gain per cent.?
Bought bricks at 1*l.* 2*s.* 7½*d.* per 1000 and sold them at 1*l.* 5*s.* 5*d.*;
at did I gain per cent.?
(7) If 5*d.* is lost in every pound; how much is lost per cent.?
If 3*s.* 1½*d.* is gained in 7*l.* 10*s.*; what is that per cent.?
(8) Bought at 3*l.* 3*s.* per cwt. and sold at 8¾*d.* per lb.; what was the
in per cent.?
Sold at 1*s.* 9¼*d.* per lb. what had cost me at the rate of 7*l.* 8*s.* per
t.; what was the gain per cent.?
(9) How must rum which cost 12*s.* per gallon be sold per pint to
in 12½ per cent.?
If I sell at 1*s.* 10¼*d.* thereby gaining at the rate of 1*s.* in 11*s.* 1½*d.*;
at was the prime cost?
(10) Bought ribbon at 9*d.* for 12 yds. and sold at 1¼*d.* per yd.; what
l I gain per cent.?
Bought rice at 10*s.* 9*d.* per cwt. and sold it for 17*s.* 11*d.*; what was
e gain per cent?

FELLOWSHIP.

Fellowship or partnership in trade or in any speculation, when two or more persons join their capital, and receive share of the gain in proportion to their stocks.

It will be readily understood that a man's share of the

gain will be in proportion to the amount of his stock and the time his stock has been employed in the business.

Fellowship may be divided into two parts, Fellowship with Time and without Time.

FELLOWSHIP WITHOUT TIME.

To find each man's stock or gain.

Rule.—As the sum of the stocks is to each man's stock, so is the whole gain to each man's share of the gain; or as the whole gain is to each man's gain, so is the whole stock to each man's stock.

EXAMPLE.—A and B join in trade. A put in 60*l.*, and B 40*l.* They gain 30*l.*; what was each man's share of the gain?

		£	
60 + 40	=	100	
As 100 : 60	::	30	
30			
100)1800			
18			A's share.
30 − 18 = 12			B's share.

In the following sums the second proves the first in each example.

EXERCISE 85.

A and B commenced partnership, A putting in 150*l.* and B 60*l.*; they gained 70*l.*; what was each man's share?

Two persons gain in trade partnership 50*l.* and 20*l.* respectively; their whole stock was 210*l.*; how much did each put in?

(2) Three persons put in business 180*l.*, 150*l.*, 100*l.*, they gain 215*l.*; what was each man's share?

If the gains of three persons in a joint business are 90*l.*, 75*l.*, and 50*l.*, and their whole stock amounted to 430*l.*; what was each man's stock?

(3) A, B, and C having divided their gains, find them to be respectively 200*l.*, 150*l.*, and 100*l.*; their whole stock was 1800*l.*; how much did each put in?

e persons' stocks are 800*l.*, 600*l.*, and 400*l.*, and their gains what was each man's share of the gain?

A, B, C, and D join in trade with 3700*l.* A put in half the B five sixteenths, C one-eighth, and D one-sixteenth; they gain 0*s.*; what is each man's share of the gain?

persons divide their gains as follows:—231*l.* 5*s.*, 144*l.* 10*s.* 7½*d.*, *s.* 3*d.*, and 28*l.* 18*s.* 1½*d.*; their whole stock amounted to 3700*l.*; as each man's stock?

Divide a gain of 914*l.* 4*s.* 9*d.* among 4 persons, in the proportion , 18, and 32.

persons join in trade. A puts in 74*l.* 18*s.* 9*d.*, B 89*l.* 18*s.* 6*d.*, 15*s.* 6*d.*, and D 479*l.* 12*s.*; they gain 914*l.* 4*s.* 9*d.*; what was an's stock?

FELLOWSHIP WITH TIME.

le.—Multiply each partner's stock by the time it has in trade; then as the sum of the products is to each ct, so is the whole gain to each man's share of the

MPLE.—A, B, and C join in trade; A puts in 40*l.* for 3 months, or 2 months, and C 30*l.* for 4 months; they gain 68*l.*; what is an's share?

40 × 3 = 120	As, 340 : 68	::	120
50 × 2 = 100	5 1		
30 × 4 = 120		5)120	
340		24 A's share.	
		20 B's share.	
		24 C's share.	

ice that the first operation in each example is proved e second, a question in fellowship with time.

EXERCISE 86.

A commences trade with 500*l.*; in 3 months B joins with 350*l.*, 9 months C with 400*l.*; at the end of 12 months they divide ns, 258*l.* 10*s.*; how much will each receive?

e persons join in trade, and in 12 months gain 258*l.* 10*s.*; A 202*l.* 10*s.*; B 106*l.* 6*s.* 3*d.*, and 40*l.* 10*s.*; what was each man's

Three farmers hire jointly a meadow for 62*l.* 18*s.*; A put in 100

sheep for 4 months; B 60 for 6 months, and C 80 for 9 months; what rent has each to pay?

A, B, and C join in trade; A puts in 123*l.* 6*s.* 8*d.*, B 111*l.*, and C 222*l.*; they gained 62*l.* 18*s.*; what is each man's share of the gain?

(3) A person begins trade with 1500*l.*; at the expiration of four months he takes a partner with 1000*l.*, and four months after another with 800*l.*; at the end of two years from his first trading they divide the profits, amounting to 1651*l.* 4*s.*; what is each partner's share?

Three merchants join in company. A puts in 385*l.* 10*s.*; B 214*l.* 3*s.* 4*d.* and C 137*l.* 1*s.* 4*d.*; they gain 1651*l.* 4*s.*; what is each man's share?

EXCHANGE.

This rule has reference to the expressing of sums of money in the value of the currency of several countries. To the merchant a knowledge of the finance of the principal states of Europe and of the world is of great importance, as it enables him to make his remittances and receive his payments most advantageously to himself.

For full explanation on this subject the learner is referred to special treatises on Exchange. It will be useful, however, to notice the following particulars.

Money is of two kinds, real and imaginary, such as coin or cash, and money of exchange, as a pound sterling, which is variable.

By the par of exchange is meant the amount of money of one country which is equal in value to a certain sum of another, regard being had to the intrinsic value of the metals of which the coins are composed.

The course of exchange is what would be allowed under or above the par of exchange. and is variable.

Agio is the difference in value between bank money and current money. Bank money is generally worth more than current by from 3 to 5 or 6 per cent.

Notice the values of the principal foreign moneys.

Country.	Name.	Equivalent.	Value.	per 1*l.* sterling British.
Amsterdam	Florin	100 centimes	1*s.* 8*d.*	11 florins 97 cents.
Berlin	Pruss. doll.	30 groschen	2*s.* $10\frac{3}{4}$*d.*	6 dols. 27 sil. grosch.
Vienna	Florins	60 kreutzers	2*s.* 0*d.* +	9 florins 50 kr.
Copenhagen	Rigs dollar	96 skillings	2*s.* $2\frac{1}{2}$*d.*	9 dols. 10 sk.
Hamburg	Mark Banco		1*s.* $5\frac{1}{2}$*d.*	13 marks $10\frac{1}{2}$ sch.
New York	Dollar	100 cents.	4*s.* 2*d.*	4 dols. 80 cents.
Madrid	1 dollar	8 reals	3*s.* $1\frac{3}{4}$*d.*	6 dols. $2\frac{3}{4}$ reals
Leghorn	Lira tosoana	100 centissimi	$7\frac{41}{50}$*d.*	30 li. 69 cents.
Lisbon	Milreis	100 reis	4*s.* 8*d.*	4 mil. 285 reis.
Frankfort	Mark	24 guld.	1*s.* $7\frac{7}{8}$*d.*	12 marks $\frac{3}{40}$ guld.
Antwerp	Florin	100 centimes	1*s.* 8*d.*	11 florins 27 cents.
Petersburgh	Rouble	100 copeks	3*s.* $1\frac{1}{2}$*d.*	6 roubles 40 copeks.

Exercise 87.

(1) How much French money is equivalent to 550*l.* 4*s.* exchange being 23 f. 45 cents per £ British?

Do this by Proportion. As, 20*s.* : 550*l.* 10*s.* 6*d.* : : 23 f. 45 c.

Exchange being at 23 f. 45 c. per £; how much English money is equivalent to 12902 f. 19 c.?

(2) Reduce 479 dollars 35 cents to British at 4*s.* $6\frac{1}{4}$*d.* per dollar.

Find the value of the dollar if 479 dols. 35 cents are equal to 108*l.* 19*s.* $5\frac{3}{5}$ British.

(3) Reduce 3524 piastres to British money at 50*d.* per piastre.

Reduce 4405 piastres to British money at 40*d.* per piastre.

(4) How much Hamburg money is equivalent to 51*l.* 17*s.* British exchange 32*s.* 8*d.* per pound sterling.

How much Hamburg money is equal to 48*l.* 16*s.* British exchange 34*s.* $8\frac{1}{2}$*d.* per pound sterling?

(5) In 790 milrees 360 rees; how much British money at 5*s.* $8\frac{1}{4}$*d.* per milree?

In 895 milrees 878 rees; how much British money at 5*s* 5*d.* per milree?

(6) Reduce 324*l.* 8*s.* 4*d.* English to Hamburg at 33*s.* 4*d.* per pound sterling?

Reduce 259*l.* 10*s.* 8*d.* English to Hamburg at 41*s.* 8*d.* per pound sterling.

PROGRESSION.

When a set of numbers increase or diminish by a common difference, they are said to be in Arithmetical Progression; if they increase or decrease by a common ratio or multiplier the series is said to be in Geometrical Progression. Thus

$1 + 3 + 5 + 7 + 9$
$9 - 7 - 5 - 3 - 1$ } are in Arithmetical progression, because they increase and decrease by a common difference.

$2 + 4 + 8 + 16 + 32$
$32 - 16 - 8 - 4 - 2$ } are in Geometrical progression, because they increase and decrease by a common ratio, 2.

ARITHMETICAL PROGRESSION.

1. *To find the sum of a series.*

Rule.—Multiply the sum of the extremes by half the number of terms.

2. *To find the common difference when the extremes and number of terms are given.*

Rule.—Divide the difference of the extremes by one less than the number of terms.

3. *The extremes and difference being given, to find the number of terms.*

Rule.—Divide the difference of the extremes by the common difference of the series; to the quotient add one for the answer.

4. *The last term, number of terms, and common difference given, to find the first term.*

Rule.—Multiply the difference by the number of terms less one, and subtract the product from the last term.

5. *The first term, number of terms, and difference given, to find the last term.*

Rule.—From the product of the common difference and number of terms, subtract the difference; to the remainder add the first term, and the sum will be the last.

6. *The difference, number of terms, and sum given, to find the first term.*

Rule.—Divide the sum of all the terms by the number of terms, and from the quotient take half the product of the difference multiplied by the number of terms less one.

In the following examples, the questions are formed from similar data, and the answer to each will be found in the same example.

Exercise 88.

(1) The first term of a series is 6, the last 216; number of terms 15; what is the sum of the series?

The first term is 6, difference 15, number of terms 15; what is the last term of the series?

(2) The first term is $2\frac{1}{4}$, difference $\frac{3}{4}$, number of terms 59; what is the last term?

The number of terms in a series is 59, difference $\frac{3}{4}$, and last term $45\frac{3}{4}$; required the first term.

(3) In an Arithmetical series, the number of terms is 9, difference $1\frac{1}{2}$, and sum $85\frac{3}{4}$; what is the first term?

The first term is $3\frac{19}{36}$, number of terms 9, and difference $1\frac{1}{2}$; what is the last term of the series?

The first term of a series is $3\frac{19}{36}$, the last $15\frac{18}{36}$, number of terms 9; what is their sum?

(4) A farrier agrees to shoe a horse at $\frac{1}{2}d.$ the first nail, $1d.$ the second, and increasing by $\frac{1}{2}d.$ each nail; as he requires 32 nails, how much will he receive for the last?

A farrier was paid for shoeing a horse at the rate of $\frac{1}{2}d.$ for the first nail, increasing by $\frac{1}{2}d.$ to the last, for which he received $1s.\ 4d.$? how many nails did he use?

(5) One boy agreed to give another for his chess board so many pins, putting 10 on the first square, increasing by 5 every square; how many would he require?

A boy agreed to give 10720 pins for a chess-board, which was at the rate of so many on the first square, increasing by 5 each time; how many did he put on the first square?

(6) A man agreed to work a month (28 days) for his master, and to be paid 2*d.* the first day, increasing each day by 1½*d.*; how much did he receive?

A man receives 2*l.* 11*s.* 11*d.* for 28 days' work, which was at the rate of 1½*d.* per day extra since he began; how much had he for his first day's work?

(7) How much does the farrier receive in Example 4, for shoeing the horse?

Admitting that he got 1*s.* 4*d.* for putting in the last of 32 nails, and that he was paid 1½*d.* more for each nail; what did he get for the first nail?

GEOMETRICAL PROGRESSION.

As has been before observed a geometrical series increases or decreases by a common ratio, that is, by being either multiplied or divided by the same number.

In a geometrical series the product of the extremes will be equal to the product of any two terms equally distant from the extremes, as in 2, 4, 8, 16, 32, 64, $2 \times 64 = 4 \times 32 = 8 \times 16$.

If the number of terms is odd, the square of the middle term is equal to the product of the extremes, or any two equally distant from the centre, as 3, 6, 12, 24, 48, square of $12 = 144 = 3 \times 48$, or 6×24.

When the extremes and common ratio are given, to find the sum of the series.

Rule.—From the product of the greater term and the common ratio, subtract the less extreme and divide the difference by one less than the common ratio.

When the extremes and number of terms are given to find the common ratio.

Rule.—Divide the greater of the given extremes by the less, and take that root of the quotient represented by the number of terms less one.

When the extremes and common ratio are given, to find the number of terms.

Rule.—Divide the greater by the less extreme, and find what power of the ratio is equal to the quotient; one more than the index of this power will give the number of terms.

When the common ratio, number of terms, and one extreme are given, to find the other.

Rule.—Multiply the less extreme, or divide the greater, by the common ratio raised to a power indicated by one less than the number of terms.

Exercise 89.

(1) The first term of a series is 4, the last 1024, ratio 2; what is the number of terms?

The common ratio of a series is 2, number of terms 9, and the first term 4; what is the last term?

(2) The first term of a series is 8, number of terms 10, and ratio 4; what is the last term?

The last term of a series is 2097152, first term 8, and ratio 4; find the number of terms.

(3) The first term of a series is $\frac{1}{4}$, ratio $\frac{1}{2}$, number of terms 7; what is the last term?

The last term is $\frac{1}{256}$, the first $\frac{1}{4}$, number of terms 7; what is the ratio?

INVOLUTION AND EVOLUTION.

Involution, or involving a number, is multiplying it so many times by itself, and the product is called a power of that number; thus $2 \times 2 \times 2 = 8$, or the cube of 3, or, as it may be expressed, the second power of 2.

To involve a number to any given power, multiply it by itself as many times as the index represents minus 1; thus 4th means that 4 must be multiplied 3 times by itself.

Bear in mind that the product of any power of a number

into any power of the same number, may be expre
adding the indices thus, $4^3 \times 4^5 = 4^8$; this considerat
facilitate the process of involving ; for instance, if
quired to raise 4 to the 16th power, put $4 \times 4 = 4^2$, 4^2
4^4, $4^4 \times 4^4 = 4^8$, $4^8 \times 4^8 = 4^{16}$, thus doing it with four
plications instead of fifteen.

The power of any compound quantity may be ex
by drawing a line over the quantity and placing th
over the last term, thus $\overline{3+4a+5b}^2$, or thus $(3+4a$
The second power of a number is called its cube.

Exercise 90.

(1)	Find the square of	16, and	4 times the square
(2)	„	27 „	9 „
(3)	„	36 „	16 „
(4)	„	105 „	9×35^2.
(5)	„	272 „	16×68^2.
(6)	„	711 „	$79^2 \times 9^2$.
(7)	„	2628 „	$657^2 \times 4^2$.
(8)	„	23652 „	$657^2 \times 36^2$.
(9)	„	47304 „	$5256^2 \times 9^2$.
(10)	„	22833 „	$177^2 \times 129^2$.
(11)	Find the cube of	16 „	square of 64.
(12)	„	64 „	„ 512.
(13)	„	324 „	$27^3 \times 12^3$.
(14)	„	2916 „	$27^3 \times 108^3$.
(15)	„	46656 „	$108^3 \times 432^3$.
(16)	Find the 4th power of	16 „	$32 \times 4^2 \times 128$.
(17)	„ 5th „	101 „	$\frac{(202)^6}{6464}$.
(18)	„ 8th „	16 „	$4096^2 \times 16^8$.
(19)	„ 12th „	25 „	$15625^3 \times 15625^2$.

To involve a fraction, raise the numerator and d
nator to the given power ; thus, the square of $\frac{3}{7} = \frac{3 \times}{7 \times}$

To find the square of a mixed number or co

quantity, bring each to a simple fraction, then proceed as before directed.

EVOLUTION.

This is the reverse of involution, inasmuch as by this rule is ascertained what number raised to a given power will produce the given quantity; thus to evolve 8 find what number multiplied by itself one or more times will produce 8; thus 8 is the cube of 2, because 2 multiplied twice by itself produces 8.

The number thus found is called the root of the given quantity; thus 4 is the square root of 16 and 2 the cube root of 8, because 4 squared produces 16, and 2 cubed, 8.

The root is expressed by a fraction placed over the quantity. The numerator of this fraction is unity and the denominator a figure representing the power to which the root must be raised to produce the given quantity; thus the square root of 4 is expressed $4^{\frac{1}{2}}$, cube root of 9, $9^{\frac{1}{3}}$, &c., or the quantity may be placed under a vinculum, as $\overline{17}^{\frac{1}{4}}$, or included within parentheses, as $(4 \times 9)^{\frac{1}{2}}$.

To find the Square Root of any number.

Rule.—Point off the given number into periods of two figures each, putting the first dot over the units place, the second over the hundreds, &c, if there are decimals put a dot over the hundreds place; another over the fourth figure from the decimal point, thus $3\dot{7}9\dot{6}4\dot{5}$, or $29\dot{1}6{\cdot}1\dot{6}1\dot{5}$.

Find the nearest root of the first period towards the left hand, and place it to the right of the given quantity; subtract its square from the first period, and to the remainder annex the next period for a dividend—find a divisor by doubling the root figure already found—see how often this is contained in the dividend, put the figure in the root and also to the right of the divisor; multiply the increased divisor by the figure last placed in the root and subtract

from the dividend, bring down the next period; double the units figure of the last divisor, and find a new root figure, which place to the right of the new divisor and proceed as before.

Squares—1, 4, 9, 16, 25, 36, 49, 64, 81.
Roots—1, 2, 3, 4, 5, 6, 7, 8, 9.

Example.—Find the square root of 181476.

Divide 181476 into periods.

Thus	18̇14̇76̇(426
	16
82	214
	164
846	5076
	5076

Explanation.—I first find the nearest square of the first period, which I find to be 16; subtract this from the first period, putting the root of the square in a quotient place to the right of the given quantity, and to the remainder adding the next period (14) for a new dividend. I then double the first root figure (4) and put it to the left of the dividend; ascertain by inspection how many times 8 is contained in 21, put the 2 in the quotient and also annex it to the trial divisor, then multiply the increased divisor (82) by 2, putting the product under the dividend (214); subtract and bring down the next period, find a new trial divisor by doubling the last figure of the previous divisor; put the quotient in the root and also to the right of the trial divisor, multiply and subtract, and proceed as before.

Exercise 91.

Find the square root of—

(1)	4096	and divide the square root of		36864	by	3
(2)	11236	„	„	44944	„	2
(3)	8836	„	„	141376	„	4
(4)	5929	„	„	148225	„	5
(5)	9801	„	„	352836	„	6
(6)	10201	„	„	499849	„	7
(7)	14884	„	„	952576	„	8
(8)	15129	„	„	1225449	„	9
(9)	79524	„	„	9622404	„	11
(10)	398927	„	„	3590343	„	3
(11)	5·76	„	„	696·96	„	11

(12)	27·3425	and divide the square root of	246·0625	by	3
(13)	·03	„	„	4·32	„ 12
(14)	·01234	„	„	·04936	„ 2
(15)	371·236	„	„	5939·776	„ 4
(16)	7946·283	„	„	71516·547	„ 3

To extract the Square Root of a fraction or mixed number.

Rule.—If the numerator and denominator be perfect squares, extract the root of each for the fractional root; sometimes when the numerator and denominator are not squares they can be made so by division; if, however, this cannot be done, bring the fractional quantity to a decimal, and find the root as before directed.

Exercise 92.

Find the square root of—

(1)	$\frac{576}{2601}$	and multiply the square root of	$\frac{144}{289}$	by	$\frac{2}{3}$
(2)	$\frac{2304}{5776}$	„	„	$\frac{256}{361}$	„ $\frac{3}{4}$
(3)	$\frac{900}{2401}$	„	„	$\frac{36}{49}$	„ $\frac{5}{7}$
(4)	$\frac{1452}{3888}$	„	„	$\frac{347}{1008}$	„ $\frac{2}{3}$
(5)	$\frac{2592}{5408}$	„	„	$\frac{144}{169}$	„ $\frac{3}{4}$
(6)	$\frac{11}{15}$	„	„	$\frac{11}{135}$	„ 3
(7)	$\frac{23}{37}$	„	„	$\frac{23}{148}$	„ 2
(8)	$3\frac{2}{5}$	„	„	$\frac{17}{125}$	„ 5
(9)	$23\frac{5}{8}$	„	„	$\frac{189}{1152}$	„ 12
(10)	$37\frac{4}{9}$	„	„	$\frac{337}{2304}$	„ 16

Note.—Carry out the decimal to six places.

Exercise 93.

(1) Find the side of a square garden which contains 1369 square feet.

A field containing 2 acres 2 roods 221 yards is to be divided into 9 allotments; what should be the side of a square equal to each?

(2) A field contains 17 acres 3 roods 15 poles 10 yards; what is the length in yards of the side of a square equal in area?

Find the side of a square containing $9597\frac{1}{12}$ square yards; also of another containing $38388\frac{1}{3}$ square yards, and add the results.

(3) What is the side in feet of a square equal in area to 15 acres 3 roods?

Find the side in feet of squares containing respectively $1\frac{3}{4}$ acre and 7 acres.

(4) How many square inches in a piece of wood 49 feet square?

How many square inches in 3 square pieces of wood, the sides of which are respectively 36, 31, and 12 feet?

CUBE ROOT.

Many rules have been proposed for extracting the Cube Root. The following is perhaps equal to any.

Point off the given number into periods of three figures each, thus 32̇5̇16̇2̇, or 27̇13̇2̇·18̇16̇3̇. Find the greatest cube of the first period, and subtract it from the period; put the root of the cube in the quotient, and bring down the next period to the remainder for a dividend. Find a trial divisor by multiplying the square of the quotient by 3; find by inspection how many times this trial divisor is likely to be contained in the dividend, and put the root in the quotient —then, to find a subtrahend, cube the last figure in the quotient; multiply all the figures in the quotient by 3 except the last, and that product by the square of the last; place this product under the cube beginning at the tens figure—one figure to the left of this, also place the product of the trial divisor and the last figure in the root—add these three results for a subtrahend which take from the dividend, and to the remainder bring down the next period, and proceed as before.

EXAMPLE.—Find the cube root of 4·410944.

$1^2 \times 3 = 3$ Trial Divisor

Cube of $6 = 216$

$3 \times 6^2 = 108$

Trial Divisor $\times\ 6 = 18$

3096 subtrahend

$16^2 \times 3 = 768$ Trial Divisor.

Cube of $4 = 64$

$16 \times 3 \times 4^2 = 768$

$768 \times 4 = 3072$

314944 subtrahend.

$\dot{4}\cdot41\dot{0}9\dot{4}\dot{4}(164$

1

3410

3096

314944

314944

Explanation.—Having pointed off the figures into periods, find the least cube to 4, this is 1, subtract, and bring down next period; now find a trial divisor by squaring the 1 and multiplying it by 3; this is found by experiment to be 6, cube the 6 and place it to the left of the dividend; under this cube and to the left place the product of 3 and 6^2, this is 108; then under this place the product of the trial divisor and the last figure; these lines being added together, the sum will be a subtrahend, which take from the dividend; proceed in the same manner with the next period till the calculation is completed.

Exercise 94.

Find the cube root of—

(1)	551368	and divide the cube root of		4410944	by	2
(2)	857375	„	„	6859000	„	2
(3)	1367631	„	„	10941048	„	2
(4)	1906624	„	„	15252992	„	2
(5)	1953125	„	„	52734375	„	3
(6)	11543·176	„	„	92345·408	„	2
(7)	3726·81945	„	„	29814·5556	„	2
(8)	219463·827	„	„	1755710·616	„	2

To extract the Cube Root of a fraction.

If the numerator and denominator be perfect cubes, extract the root of each for the fractional root. Sometimes when the numerator and denominator are not cubes they can be made so by division; if, however, this cannot be done, reduce to a decimal and extract the root. In any case it is always best to find the greatest common measure, and reduce before extracting the root.

Find the cube root of—

(9)	$\frac{1000}{9261}$	and multiply the cube root of		$\frac{125}{343}$	by	$\frac{2}{3}$
(10)	$\frac{729}{4096}$	„	„	$\frac{27}{64}$	„	$\frac{3}{4}$
(11)	$\frac{4608}{61731}$	and divide the cube root of		$\frac{36864}{61731}$	„	2
(12)	$\frac{4459}{8656}$	„	„	$\frac{285376}{130393}$	„	$\frac{3}{4}$
(13)	$27\frac{3}{7}$	„	„	$219\frac{3}{7}$	„	2
(14)	$45\frac{4}{5}$	„	„	$1236\frac{3}{5}$	„	3

(14) If a cistern measures 9 feet each way, how many cubic inches does it contain?

Into 27 cisterns each measuring 36 inches every way, how many cubic inches of water could be put?

(16) In a cube, the side of which is 18 feet, how many cubic inches ?

In 3 cubes, the sides of which are respectively 15, 12, and 9 feet, how many cubic inches ?

(17) If a cubical cistern contains 1860862 inches, what is the length in feet of one of its sides ?

A cistern contains 14886896 cubic inches, what is half the length in feet of one of its sides ?

MISCELLANEOUS QUESTIONS.

Exercise 95.

(1) What will 9 pieces of cloth, each $20\frac{1}{2}$ yards, cost, at 2*s.* $9\frac{1}{2}$*d.* per yard ; also 36 pieces, each $10\frac{1}{4}$ yards, at 1*s.* $4\frac{3}{4}$*d.* per yard ?

(2) Multiply 12 lbs. 7 oz. 3 drs. by 10 ; and divide 12 cwt. 25 lbs. 6 oz. 10 drs. by 11.

(3) Reduce 179 tons to lbs. ; and £417 13*s.* 4*d.* to farthings.

(4) What is the 92nd part of 17 times $10\frac{3}{4}$ guineas ? also the 23rd. part of £5 12*s.* $10\frac{1}{2}$*d.* multiplied by $8\frac{1}{3}$.

(5) Add together £37 15*s.* and $29\frac{3}{4}$ guineas ; also add $15\frac{1}{2}$ guineas, $7\frac{3}{4}$ guineas, $30\frac{3}{4}$ guineas, and £12 5*s.* 9*d.*

(6) Square the 15th part of 435 ; and divide 34481 by the difference of the squares of 20 and 21.

(7) Reduce 5685 lbs. to tons ; and multiply 3 cwt. 1 qr. 15 lbs. by 15.

(8) Value 924 books at 5*s.* 6*d.* per dozen ; also 1386 books at 3*s.* 8*d.* per dozen.

(9) Add together 13 lbs. 11 oz. 7 dwts. 17 grs., 29 lbs. 9 oz. 10 dwts. 21 grs., and 15 lbs. 9 oz. 16 dwts. 23 grs. ; and subtract 37 lbs. 3 oz. 14 dwts. 5 grs. from 96 lbs. 10 oz. 9 dwts. 18 grs.

(10) How many books at 9*d.* each can I buy with 11 guineas ? and how many at $6\frac{3}{4}$*d.* each with £8 13*s.* 3*d.* ?

(11) If 17 lbs. cost 10*s.* $9\frac{3}{4}$*d.*, what will 89 lbs. cost ? and if $11\frac{1}{3}$ lbs. cost 3*s.* $7\frac{1}{4}$*d.*, what will 178 lbs. cost ?

(12) How much will 10500 bricks cost at 32*s.* 3*d.* per thousand ; also 15750 bricks at 21*s.* 6*d.* per thousand ?

(13) Take the sum of $4\frac{1}{2}$ and $5\frac{1}{3}$ from $12\frac{1}{4}$; and divide $1\frac{1}{2\frac{1}{8}}$ by $\frac{3}{7}$.

(14) Find the sum, difference, product, and quotient of $\frac{1}{2}$ and $\frac{2}{5}$ fractionally and decimally, reducing the fractional results to decimals.

(15) Required the value of 317 lbs. of tea at 3*s.* 6*d.* per lb.; also 1 cwt. 1 qr. 18 lbs. 8 oz. at $5\frac{1}{4}d.$ per oz.

(16) Reduce 179 tons 17 cwt. 1 qr. 10 lbs. to drams; and £1074 : 0 10*s.* 8*d.* to farthings.

(17) What will 28 pieces of cloth, each 27 yards, cost at 10*s.* $11\frac{1}{2}d.$ per yard? also 18 pieces, each 84 yards, at 5*s.* $5\frac{3}{4}d.$?

(18) Value 9 tons at £7 16*s.* 6*d.* for 1 cwt. 3 qrs. 7 lbs.; also 18 tons at £2 12*s.* 2*d.* for 2 cwt. 2 qrs. $24\frac{1}{2}$ lbs.

(19) From the cube of 25 take the square of 125; and from the square of 78 take 4 times the square of 39.

(20) How much will pay 29 men for a week's work at 2*s.* 9*d.* per day? and what is the value of 33 dozen pairs of gloves at 1*s.* $2\frac{1}{2}d.$ per pair?

(21) If 9 men eat 40 loaves in a week, how long will 220 loaves last them? and how many weeks are there in 55440 minutes?

(22) How much sugar at 5*d.* per lb. can I have for 3 cwt. 1 qr. 10 lbs. of tea at 3*s.* $6\frac{1}{2}d.$? and reduce 813824 drams to tons.

(23) Divide £9 by $7\frac{2}{3}$; and £39 by $33\frac{3}{5}$.

(24) What should I gain on 395 books by buying them at 3*s.* 9*d.* per dozen and selling them at 10*s.* per score?

Required the value of 790 yards of ribbon at the rate of 8 yards for 9*d.*

(25) Multiply 3 cwt. 1 qr. 10 lbs. 15 oz. by 39; and reduce 3743376 drams to tons.

(26) Bought sugar at £2 10*s.* per cwt. and sold it at $6\frac{1}{2}d.$ per lb., what did I gain per cwt.?

Bought 70 lbs. of sugar for £1 11*s.* 3*d.* and sold 16 lbs. for 8*s.* 8*d.*, what was the rate of gain per cwt.?

(27) Required the value of 29 reams of paper at $\frac{3}{4}d.$ per sheet; also $48\frac{1}{8}$ gross of books at 2*s.* 6*d.* per score.

(28) What is the interest of £157 2*s.* 6*d.* for 1 year at $2\frac{1}{2}$ per cent.? also of £15 14*s.* 3*d.* for 4 years at $6\frac{1}{4}$ per cent.?

(29) Find the interest of £215 10*s.* for 35 days at 4 per cent.; also of £107 15*s.* for 42 days at $6\frac{3}{8}$ per cent.

(30) Reduce 33690384 drams to tons; and multiply 10 cwt. 4 lbs. 13 oz. by 117.

(31) What fraction is 16*s.* $5\frac{1}{2}d.$ of £8 16*s.* 9*d.*? also 3 lbs. 1 oz. 6 drs. of 24 lbs. 6 drs.?

(32) From $729\frac{5}{8}$ guineas take $573\frac{3}{4}$ crowns; and from $1259\frac{1}{4}$ half-guineas take $307\frac{1}{2}$ half-crowns.

(33) Divide 10 tons by 27, and 25 tons by $67\frac{1}{2}$; and bring the quotient in each case as low as drams.

(34) Gave 2*s.* 8*d.* for a yard of cloth and sold it for 3*s.*, what did I gain per cent.?

What is gained per cent. by selling at 6*s.* 9*d.* what was bought for 6*s.*

(35) How long would a person be counting a million of sovereigns at the rate of 40 a minute ? and divide 503 days 11 hours 20 minutes by 29.

(36) What income shall I get by investing £1500 in the 4 per cents. at 80 ? and find the interest of £4500 for 73 days at $8\frac{1}{3}$ per cent.

(37) Find the greatest common measure of $\frac{1281}{1799}$ and $\frac{1519}{2471}$.

(38) Value 37 bales of cotton, averaging 150 lbs. each, at 1*s.* $2\frac{1}{8}d.$ per lb. ; also value $2118\frac{3}{4}$ yards of silk at 3*s.* 1*d.* per yard.

(39) How many people can receive 5*s.* 6*d.* each out of £5 10*s.* 9*d.* ? and how many may receive 12*s.* $4\frac{1}{2}d.$ each out of £12 9*s.* $2\frac{1}{4}d.$?

(40) A field is divided into 25 allotments, each containing 35 perches, what is the size of the field ? also divide 36 acres 15 poles by $6\frac{3}{5}$.

(41) Reduce £791 16*s.* $11\frac{1}{2}d.$ to farthings ; and 339 tons 7 cwt. 1 qr. 2 lbs. to lbs.

(42) Divide the product of $3\frac{1}{4}$ and $2\frac{3}{8}$ by the difference of $1\frac{1}{2}$ and $\frac{3}{5}$, and reduce the result to a decimal.

Multiply 2·375 by 3·25 and divide the product by 1·125.

(43) Value 2 cwt. 3 qrs. 10 lbs. at £3 15*s.* for 1 cwt. 1 qr. 12 lbs. ; also value 1 cwt. 1 qr. 14 lbs. at £5 12*s.* 6*d.* for 1 cwt. 2 lbs.

(44) What wages must a man receive for 57 days' service at £50 3*s.* 4*d.* a year ? also for 133 days at £21 10*s.* a year ?

(45) Gave 36*s.* for a hogshead of beer and retailed it, after putting in 9 gallons of water, at 2*d.* a pint, what did I gain on the whole ?

Bought a hogshead of wine for £18 18*s.*, of which 8 gallons leaked away, what was my loss ?

(46) What is the square root of 77841 ? and divide ·58032 by ·00208.

(47) Divide the product of ·0187 and $3\frac{5}{8}$ by the product of 11·3 and 1·185 ; also divide the product of ·04363 and $8\frac{11}{24}$ by the product of $26\cdot\dot{4}$ and 2·765.

(48) If a house rented at £80 is taxed £3 6*s.* 8*d.*, how much is that per £1 ; and another rated at £374 2*s.* 6*d.* is taxed £15 11*s.* $9\frac{1}{4}d.$, how much is that per £1 ?

(49) What discount must be allowed at $2\frac{1}{2}$ per cent. on £36 8*s.* 4*d.*? and on £54 12*s.* 6*d.* at $1\frac{3}{8}$ per cent. ?

(50) Value 7 acres 3 roods 10 poles at £59 10*s.* per acre ; also 19 acres 2 roods 5 poles at £23 16*s.* per acre.

(51) How many quantities, each 3 oz. 6 drs., can be cut out of 2 cwt. 3 qrs. 4 lbs. of cheese ? also how many portions, each 7 oz. 14 drs., *can be cut out of* 6 cwt. 2 qrs. ?

(52) How much paper 1 foot 9 inches wide will cover the walls of a square room 17 feet 6 inches long and 10 feet 9 inches high ?

How much in length that is 4 feet 1 inch wide is equal to 490 feet long and 3 feet 7 inches wide ?

(53) Square the difference of $2\frac{1}{3} \times 3\frac{1}{4}$ and $\frac{5}{6} \times \frac{3}{5}$.

Square 7·27083 and reduce the result to a vulgar fraction.

(54) Value 598 lbs. of gold at £3 17*s.* 11½*d.* per oz. ; also find the value of 3742 yards of cloth at 12*s.* 5½*d.* per yard.

(55) From $353\frac{7}{8}$ guineas take $597\frac{3}{10}$ half-crowns ; also from $901\frac{3}{4}$ half-guineas take £176 10*s.* 3*d.*, and give the results in pounds, &c.

By how much does the square of 5796 exceed the cube of 229 ? and what number added to 8 times the cube of 114·5 will make it equal to 36 times the square of 966 ?

(57) Divide the square of 5796 by the cube of 229 ; also divide the square of 46368 by the cube of 916 to six places of decimals.

(58) What is the interest of £715 10*s.* 6*d.* for $17\frac{3}{4}$ years at $2\frac{3}{4}$ per cent. ? also of £1967 13*s.* 10½*d.* for 4 years at £4 8*s.* 9*d.* per cent. ?

(59) What is gained per cent. by buying at 25*s.* and selling at 33*s.* 6*d.* ? and what is gained by buying at £2 16*s.* 3*d.* and selling at £3 15*s.* 4½*d.* ?

(60) What is gained per cent. by buying at 7*s.* 6*d.* and selling at 8*s.* 5½*d.* ? and what is lost by buying at 3*s.* 4*d.* and selling at 2*s.* 11*d.* ?

(61) Reduce 315 quarters to pints and 5 cwt. 3 qrs. 14 lbs. to drams.

(62) Find the square root of 222784, and the cube root of 105154048.

(63) How many seconds in $576\frac{3}{4}$ years ? and how many minutes in 34575 years ?

(64) Multiply 3 qrs. 15 lbs. 10 ozs. 13 drs. by 343 ; and divide 5235 tons 3 cwt. 0 qrs. 19 lbs. 15 oz. 13 drs. by 343.

(65) Find the square root of 89·1136, and the cube root of 841·232384.

(66) In 79 acres, how many yards ? and in £398 5*s.* 10*d.*, how many farthings ?

(67) Multiply £37 9*s.* 10½*d.* by 693 ; and divide £922402 9*s.* 9¾*d.* by $35\frac{1}{2}$.

(68) One merchant fails for £2350, his assets are £73 8*s.* 9*d.* ; another fails for £7637 10*s.*, and his assets are £238 13*s.* 5½*d.* ; what does each pay in the pound ?

(69) Find the interest of £357 10*s.* for 120 days at $4\frac{1}{2}$ per cent. ; also of £178 15*s.* for 180 days at 6 per cent.

(70) Value 120 reams of paper at 1*s.* 6*d.* per quire ; also 280 reams at 14 sheets for 4½*d.*

(71) Value 792 hats at 5*s*. 9*d*. ; and 276 yards of velvet at 16*s*. 6*d*. per yard.

(72) Divide the square of $3\frac{1}{2}$ by $5\frac{1}{8}$; also divide 30·625 by 12·8125, and express the result as a vulgar fraction.

(73) Reduce $\frac{3}{4}$ of $1\frac{1}{8}$ of $\frac{\frac{1}{2}}{\frac{3}{4}}$ to a simple fraction, and reduce the result to a decimal.

Multiply the product of ·75 by 1·125 by the quotient of 5 divided by ·75.

(74) Value 7 tons 11 cwt. of soap at $5\frac{1}{4}d$. per lb. ; also 3 tons 4 cwt. 2 qrs. 24 lbs. of gum at 1*s*. $0\frac{1}{4}d$. per lb.

(75) Reduce 796 half-crowns to florins and 59700 pence to crowns.

(76) What is the greatest common measure of $\frac{2234}{7918}$ and $\frac{158}{1122}$?

(77) Find a fourth proportional to $25\frac{5}{11}$, $32\frac{8}{11}$, and 11 ; also a third proportional to $17\frac{1}{2}$, $22\frac{1}{2}$, and $18\frac{9}{16}$.

(78) What is the value of $\frac{5}{16}$ of a ton ? and multiply 7 cwt. 2 qrs. by ·83$\dot{3}$?

(79) Square 4913 ; and cube 289.

(80) How many pairs of gloves at 1*s*. $9\frac{3}{4}d$. must be given in exchange for 36 pieces of silk, each $17\frac{1}{2}$ yards, at 2*s*. 6*d*. per yard ?

How many neckties at 1*s*. $2\frac{1}{2}d$. must be given for 9 pieces of cloth, each 70 yards, at 1*s*. 8*d*. per yard ?

(81) Multiply the cube root of 1953125 by the square root of 10609 ; and divide 20·046375 by ·001557.

(82) Find the value of 49 loads of hay at 1*s*. $6\frac{1}{2}d$. per truss ; also value 294 dozen of brooms at 15*s*. 5*d*. per score.

(83) If the 6*d*. loaf weighs 5 lbs. 8 oz. when wheat is at 45*s*. per quarter, what should it weigh when wheat is 36*s*. per quarter ?

Reduce 5·5 × 1·25 oz. to its proper quantity.

(84) Received £24 15*s*., amount of commission at $2\frac{1}{2}$ per cent., what was the value of goods sold ?

What principal will gain £222 15*s*. in 5 years at $4\frac{1}{2}$ per cent. ?

(85) Divide the square root $\frac{27}{48}$ by the cube root of $\frac{207}{1375}$; and find the cube root of 1·953125.

(86) What number divided by ·1735 will give ·0129 as a quotient ; and take the square of ·0353 from ·00348424.

(87) How long must £500 be at interest to gain £123 15*s*. at $4\frac{1}{2}$ per cent. ?

How long must £770 be at interest to amount to £981 15*s*. at 5 per cent. ?

(88) Giving each man 9*d*., each woman 6*d*., and each boy 4*d*., among how many may £8 9*s*. 5*d*. be distributed, and of each an equal *number* ?

Divide £31 15*s.* $3\frac{3}{4}$*d.* among poor people, giving each 5*s.* $11\frac{1}{4}$*d.*, how many can receive it?

(89) Reduce £2 3*s.* 8*d.* to a fraction of £7 9*s.* 6*d.*; also 3 cwt. 2 qrs. 1 lb. to the fraction of 12 cwt. 1 lb. 8 oz.

(90) Reduce 3*s.* $10\frac{1}{2}$*d.* to the decimal of a pound; and 6 oz. 19 dwts. 12 grs. to the decimal of 3 lbs. (Troy).

(91) Multiply 3 cwt. 1 qr. 10 lbs. by $3\frac{3}{4}$; and divide 45 tons 1 cwt. 2 qrs. 12 lbs. by 72.

(92) Divide 17 tons by 29 as low as drams; also divide 38 tons 5 cwt. by $65\frac{1}{4}$.

(93) Value 2716 bonnets at 10*s.* 9*d.*; also 8148 caps at 3*s.* 7*d.* (do both by practice).

(94) What cost 17 cwt. 2 qrs. 10 lbs. of tea at the rate of 3 cwt. 1 qr. 5 lbs. for 30 guineas?

Value 8 cwt. 3 qrs. 5 lbs. at the rate of 5 cwt. 1 qr. 27 lbs. for £105.

(95) Simplify $\frac{\frac{2}{3} \text{ of } 3\frac{1}{4}}{\frac{1}{7} \text{ of } \frac{3}{2\frac{1}{2}}}$, and reduce the result to a decimal.

Divide 2·166 by ·1714285, and prove the last example.

(96) How many times is ·0125 contained in $\frac{1}{2}$ of $2\frac{1}{3}$ of $3\frac{1}{4}$?

Reduce $2\frac{25}{33}$ to a decimal, and divide the result by ·0090.

(97) What number cubed is equal to the square of 704969?

What is the difference in the square roots of 564680169 and 250968964?

(98) What is the commission on £3765 16*s.* 8*d.* at £3 6*s.* 9*d.* per cent.?

What is due to a broker for effecting sale of £33892 10*s.* stock at 7*s.* 5*d.* per cent.?

(99) Paid £39 17*s.* 6*d.* for insuring goods valued at £3190, what was the premium?

Received £41 19*s.* $0\frac{3}{4}$*d.* as interest on £895 for $3\frac{3}{4}$ years, what was the per centage?

(100) Divide £5080 among four persons in the proportion of 4, 6, 9, and 13.

Four persons engage in trade together—A's stock was £694 1*s.* 8*d.*, B's £1041 2*s.* 6*d.*, C's £1561 13*s.* 9*d.*, and D's £2255 15*s.* 5*d.*; they gain £5080, what was each man's share of the gain?

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